# Chapter 9
# An Evolutionary Cul-de-Sac

*Gerontomorphosis cannot lead to radical changes and new departures;*
*it can only carry an already specialized evolutionary line one more step further*
*in the same direction—as a rule into a dead end of the maze.*

Arthur Koestler

As evolution during the past 5,000 years has been more mental than biological, our behaviour as human beings has been primarily determined by our learning. For the most part, we are not conscious of how we learn. We go about our daily lives focusing attention on building a home, growing food, painting pictures, earning a living, and so on and so forth. However, over the years, scientists, mathematicians, logicians, and philosophers have attempted to formalize our learning processes in order to ensure that our reasoning is valid, that it produces an accurate picture of the world we live in. For if our reasoning is invalid, then we must inevitably be deluded.

These principles of scientific inquiry and logical reasoning have not been static over the years. Like all our learning, they have undergone an evolutionary development, sometimes leading to a crisis when the assumptions on which these rational processes take place have proved to be unreliable. That is the situation in the world today. By denying the role that Life plays in our creative evolutionary processes, traditional Western reasoning has reached an evolutionary cul-de-sac; there is nowhere else for it to grow and develop.

In the preface to Part I, 'Integral Relational Logic', I pointed out that, in general, when evolution reaches a dead end, it needs to backtrack to an earlier stage in its development and continue growth from there. For such a pædomorphic process is rejuvenating, enabling evolution to continue on its relentless path towards Wholeness. IRL is just such a holistic science of reason that enables us to fly like the birds in the sky, without any restrictions on what we learn or how we learn it. This is because it is based on profound, abstract structures that are equally applicable in all domains of learning.

So let us use IRL to look at the evolution of scientific method and mathematical logic to see how we have reached the impasse that we are in today and what we might be able to do to unblock and heal our split minds. We can liken this situation to a dam wall holding back immense potential energy behind it. If this energy is to be released for the benefit of us all, we need to demolish the wall, either bit by bit or in one great explosion, rather like a tsunami sweeping away all before it. This chapter takes the former approach, examining some of the concrete slabs in the wall that prevent us from making peace possible by healing the split between Western reason and Eastern mysticism and between logic and psychology.

The central point here is that we can only act consciously in the world with full awareness of what we are doing through self-inquiry. Historically, the mystics have led the way in this endeavour through their meditative and contemplative techniques. The mystics have thus shown us the way to the Truth, being far more scientific than those we call scientists claim to be. For scientists avoid looking at their inner worlds in the belief that an external, objective reality exists independent of a knowing being. We urgently need to correct this misconception.

We first look at the way that mathematics, the language of science, and logic, the science of mind and reason, has also reached an evolutionary cul-de-sac. For centuries, mathematics was seen as the one discipline in which certainty and irrefutable truth could be found. But when mathematicians sought certainty in mathematics in the first half of the twentieth century, it eluded them, basically because they were using linear thought processes in the horizontal dimension of time, not nonlinear, initiated by Life in the vertical dimension.

We next take a brief overview of the way scientific method has evolved over the years, showing that it has reached a dead end because it cannot explain why the pace of technological development is accelerating exponentially, why evolution is currently passing through its accumulation point in systems theory terms. As a result, we are managing our business affairs having very little understanding of what we are doing.

As neither mathematical logic nor scientific method can lead us to Wholeness and the Truth, it is not surprising that physicists' theories of what they consider to be the universe have reached an evolutionary blind alley. We look at some of the consequences of this and how we can resolve the incompatibilities found by the physicists in the last century within the context of the Universe viewed as Consciousness. In so doing we can also bring Life back to science, which the biologists are doing their best to deny.

## The loss of certainty

Although it is not necessary to know anything about the history of mathematics and logic to be an awakened individual in an awakened society, the hidden assumptions of these subjects lie deep in the collective unconscious of Western civilization. So if we are to be free of this conditioning, to end our sleepwalking habits, it is vitally important that we bring our suppositions into awareness so that they can be examined in the full light of Consciousness.

Now because the ancient Greeks were living some two thousand years after the dawn of history, the idea that time is linear, with a past and future, was well established in their culture. So it was natural for them to assume that both the chain of cause and effect and human reasoning are also linear. In the case of the former, Aristotle reasoned that there must be an unmoved mover that brings all motion into effect.[1] This meant that Aristotle was not aware that the unmoved mover is the Absolute and that all change arises through the effect of Life or the Logos arising directly from our Divine Source in the vertical dimension of time.

The one outstanding exception to this state of ignorance in ancient Greece was Heraclitus, the mystical philosopher of change, who lived between 540 BCE and 475 BCE, about 150 years before Aristotle. Very little of Heraclitus' writings survive, only fragments, some of which are quotations by others, not direct quotes. But the fact that these fragments exist at all shows that his was a voice that could not be ignored.[2]

Heraclitus was essentially a both-and thinker, grounded in Wholeness, as this fragment shows quite clearly: "God is day and night, winter and summer, war and peace, satiety and want."[3] As a mystic, he was also acutely aware of the primal energy that makes manifest the entire world of form. He called this the *Logos*, which Lao Tzu called *Tao*, the *Upanishads* and *Vedas rit*, and Shankara *brahma*, as Osho points out.[4]

He was very well aware that few of his contemporaries understood what he meant by *Logos*, as these fragments indicate: "Although the Logos is eternally valid, yet men are unable to understand it—not only before hearing it, but even after they have heard it," "Yet, although the Logos is common to all, most men live as if each of them had a private intelligence of his own," and "Although intimately connected with the Logos, men keep setting themselves against it."

This lack of understanding of the mystical meaning of *Logos* led Heraclitus to say, "Eyes and ears are poor witnesses for men if their souls do not understand the language."[5] So his contemporaries called him 'the Obscure' and Aristotle accused him of not reasoning.[6] Aristotle did not understand that Wholeness is the union of all opposites, and as such, is the basis of all reason. Rather Aristotle said, "it is impossible for anyone to suppose the same thing is and is not, as some imagine that Heraclitus says."[7] Rather more precisely, Aristotle asserted, not in *Organon*, but in *Metaphysics*, "It is impossible for the same attribute at once to belong and not to belong to the same thing and in the same relation,"[8] which is the seventh pillar of unwisdom underlying Western civilization.

From the point of view of Wholeness, it does not matter whether this Law of Contradiction, the fundamental law of Western thought, is true or not. Indeed, as mathematicians and logicians discovered in the twentieth century, this divisive law is not universally true. So it is time to put the Logos back into logic, which neither Aristotle nor any of his successors have been able to do, and thereby to put logic back into psychology, where it properly belongs. Indeed, it was not until the invention of the stored-program computer in the middle of the last century that we were able to discover the error of our ways. As Heraclitus said, "We should let ourselves be guided by what is common to all," that is by the Logos.[9]

## Beginnings of Western reason

However, Aristotle did not do this. For Aristotle, time was linear, which is well demonstrated in the *Organon*, the five books that constitute the foundations of Western reason called *Categories*, *On Interpretation*, *Prior Analytics*, *Posterior Analytics*, and *Topica*. In the third of these books, he developed the syllogism by examining structures consisting of three propositions (also called statements or sentences), called the major premise, minor premise, and conclusion, respectively. Each proposition has two terms called the subject and predicate.

Aristotle developed some rules for determining which of these structures leads to a valid conclusion and which do not. In doing this, he naturally used Integral Relational Logic in studying the various attributes of the propositions. For instance, the terms in a proposition are related to each other in four different ways, as the relation in Table 9.1shows. The diagrams come from Leonhard Euler (1707–1783), which were extended in 1880 by John Venn (1834–1923), known today as Euler and Venn diagrams.

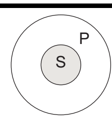
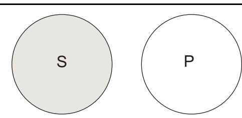
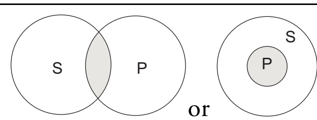
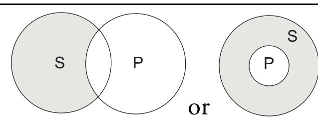
| **Class name** | *Syllogistic propositions* | | |
|---|---|---|---|
| **Attribute name** | *Name* | *Form* | *Diagram* |
| **Attribute values** | A | All S are P |  |
| | E | All S are not P |  |
| | I | Some S are P |  |
| | O | Some S are not P |  |

Table 9.1: *Forms of syllogistic propositions*

These propositions have three pairs of attributes that characterize the propositions, as the following relation shows. Any two of these attributes uniquely defines the proposition. So we could call them defining attributes, with the third being derivable from the other two, as in Table 9.2.

| **Class name** | *Syllogistic propositions* | | | |
|---|---|---|---|---|
| **Attribute name** | *Name* | *Universality* | *Positivity* | *Symmetry* |
| **Attribute values** | A | universal | positive | asymmetrical |
| | E | universal | negative | symmetrical |
| | I | particular | positive | symmetrical |
| | O | particular | negative | asymmetrical |

Table 9.2: *Characteristics of syllogistic propositions*

Aristotle called the symmetrical propositions convertible because they are equivalent when the terms are interchanged. A and E are also convertible into weaker forms, I and O, respectively. Furthermore, if we assume Aristotle's Law of Contradiction to be true, A and O and E and I are contradictory; they exclude each other.

One other property of these propositions relates to the terms in the proposition, rather than the propositions themselves. A term is distributed if, in some sense, it refers to all entities with the particular property (called a class), otherwise it is undistributed. The subject of universal propositions and the predicate of negative propositions are distributed, as shown in the relation in Table 9.3.

| **Class name** | *Syllogistic propositions* | | |
|---|---|---|---|
| **Attribute name** | *Name* | *Subject* | *Predicate* |
| **Attribute values** | A | distributed | undistributed |
| | E | distributed | distributed |
| | I | undistributed | undistributed |
| | O | undistributed | distributed |

Table 9.3: *Distributed properties of syllogistic propositions*

The terms of the three propositions of the syllogism are related to each other in two ways:

a) One term is common to the major and minor premises; it is called the middle term (M).
b) The predicate (P) of the conclusion is the major term of the syllogism and the subject (S) is the minor term, because they are the non-middle terms in the major and minor premises, respectively.

As propositions are one of four types and as there are three propositions in each syllogism, there are $4^3$=64 different syllogistic forms, called moods. These are naturally called AAA, AAE, AAI, etc.

In addition, the syllogism can have one of four figures, depending on whether the middle term is the subject or predicate in the major and minor premises, shown in Table 9.4. (Curiously, for some reason, Aristotle only recognized three of these figures; the fourth was not discovered until the Middle Ages.) There are thus 64*4=256 possible syllogisms in total.

| **Class name** | *Syllogistic figures* | |
|---|---|---|
| **Attribute name** | *Name* | *Figure* |
| **Attribute values** | I | $\begin{array}{cc} M & P \\ S & M \\ \hline S & P \end{array}$ |
| | II | $\begin{array}{cc} P & M \\ S & M \\ \hline S & P \end{array}$ |
| | III | $\begin{array}{cc} M & P \\ M & S \\ \hline S & P \end{array}$ |
| | IV | $\begin{array}{cc} P & M \\ S & M \\ \hline S & P \end{array}$ |

Table 9.4: *Syllogistic figures*

Aristotle examined each mood and figure in turn to determine whether it was valid or not. He then derived a number of common properties of these syllogisms, which can be called rules of deduction. I reverse this process here. These are the rules that Aristotle discovered:

1. Relating to premises irrespective of conclusion or figure
   a) No inference can be made from two particular premises.
   b) No inference can be made from two negative premises.
2. Relating to propositions irrespective of figure
   a) If one premise is particular, the conclusion must be particular.
   b) If one premise is negative, the conclusion must be negative.
3. Relating to the distribution of terms
   a) The middle term must be distributed at least once.
   b) A predicate distributed in the conclusion must be distributed in the major premise.
   c) A subject distributed in the conclusion must be distributed in the minor premise.

This leaves us with 19 valid syllogisms, found by Aristotle and his successors:

First figure: AAA, EAE, AII, EIO
Second figure: EAE, AEE, EIO, AOO
Third figure: AAI, EAO, IAI, AII, OAO, EIO
Fourth figure: AAI, AEE, IAI, EAO, EIO

Students in the Middle Ages were expected to know all these by heart. For instance, the statutes of the University of Oxford in the fourteenth century included this rule: "Bachelors and Masters of Arts who do not follow Aristotle's philosophy are subject to a fine of 5s for each point of divergence, as well as for infractions of the rules of the *Organum*."[10] Not surprising therefore that they needed a mnemonic to remember this rather arbitrary set of letters:

Barbara, Celarent, Darii, Ferioque
Cesare, Camestres, Festino, Baroco
Darapti, Felpaton, Disamis, Datisi, Bocardo, Ferison
Bramantip, Camenes, Dimaris, Fesapo, Fresison.[11]

These syllogisms can be further reduced because propositions E and I are symmetrical; the terms in these propositions can be interchanged. Also, some syllogisms are weak forms of stronger ones. This means that there are just eight core syllogisms out of the 256 candidates that we started with: AAA (I), AII (I), EAE (I), EIO (I), AOO (II), AAI (III), EAO (III), and OAO (III).

Just as Aristotle did not begin at our Divine Source in developing his logic, neither did Euclid, who lived about 200 BCE, about a century after Aristotle, in laying down the fundamental principles of mathematical proof in *The Elements*. Although many of the theorems in *The Elements* were not new, what is now a three-volume work, studied by all educated people until the twentieth century, was the first attempt to create a systematic approach to mathematical theorems.

Euclid began his first book of mathematical reasoning with twenty-three definitions, five postulates, and five common notions, which today we would call axioms.[12] To Euclid, these were self-evident truths, although he doesn't explicitly say this. Today, axioms are more likely to be regarded as assumed truths, none of which is, of course, the Truth, which cannot be expressed in symbols of any sort.

Nevertheless for more than two millennia, mathematics was regarded as a way of leading to certain knowledge about the world we live in. As Morris Kline tells us in *Mathematics: The Loss of Certainty*, "Mathematics was regarded as the acme of exact reasoning, a body of truths in itself, and the truth about the design of nature".[13] Maybe many still believe in this view of mathematics. Yet despite its great success in making predictions about the physical universe and the many theorems it has discovered, mathematics, as it has evolved today, falls far short of this ideal picture. Nevertheless, mathematics is regarded as the archetype of conceptual clarity, as this well-known joke illustrates:

> An astronomer, a physicist, and a mathematician (it is said) were holidaying in Scotland. Glancing from the train window, they observed a black sheep in the middle of the field. 'How interesting,' observed the astronomer, 'all Scottish sheep are black!' To which the physicist responded, 'No, no! Some Scottish sheep are black!'. The mathematician gazed heavenward in supplication, and then intoned, 'In Scotland there exists at least one field, containing at least one sheep, at least one side of which is black.'[14]

So what went wrong? Why is it that the pursuit of conceptual clarity has not led to conceptual integrity? Why is it that the great body of truths that mathematics has discovered do not add up to the Truth? Why doesn't mathematical logic tell us what it truly means to be a human being, in contrast to the other animals and machines, like computers? The answer is very simple. Mathematics, as it has been practiced over the years, is not based on the fundamental principle of mapmaking, "Accept everything; reject nothing".[15] Rather, mathematics is based on the first and seventh pillars of unwisdom, in particular. It is not surprising therefore that mathematics has no solid foundation and so cannot possibly lead us to Wholeness and the Truth.

## The laws of thought

Logic, the science of reason, followed an independent path for over two thousand years, based primarily on Aristotle's syllogism. Mathematics and logic were seen as being quite distinct from each other.

However, in 1854, George Boole (1815–1864) wrote a seminal book called *An Investigation of the Laws of Thought on Which Are Founded the Mathematical Theories of Logic and Probabilities*, a development of an earlier pamphlet called *The Mathematical Analysis of Logic: Being an Essay towards a Calculus of Deductive Reasoning*, published in 1847. Here is the first sentence of this former work: "The design of the following treatise is to investigate the fundamental laws of those operations of the mind by which reasoning is performed," with the purpose of exploring "the nature and constitution of the human mind".[16]

Apparently, he had been moved to do so by a mystic experience he had had when seventeen in early 1833, when the thought flashed through him as he was walking across a field that logical relations could be expressed in symbolic or algebraic form. This was an idea that Gottfried Wilhelm Leibniz had explored during the last third of the seventeenth century,[17] although Boole was unaware of this at the time. By thus explaining the logic of human thought, he felt it possible to delve analytically into the spiritual aspects of man's nature. As Desmond MacHale, his biographer, tells us, "Boole referred to the incident many times in later life and seems to have regarded himself as cast in an almost messianic role."[18]

In preparation for these seminal books, in 1844, Boole wrote a paper called 'On a General Method in Analysis' published in the *Philosophical Transactions of the Royal Society of Lon-*

*don*.[19] At the time, there was a movement towards ever greater generalization in mathematics. For instance, the concept of number had evolved from all positive integers, to all integers, including zero, to rationals, reals, and complex numbers of the form $a + ib$, where $i = \sqrt{-1}$. However, while it was known in 1830 that while complex numbers could be used to represent vectors in two-dimensional Euclidean space, no way had been found to extend such numbers to three dimensions. In trying to solve this problem, in 1843, William R. Hamilton (1805–1865) discovered what he called quaternions of the form $a + ib + jc + kd$, where $i = j = k = \sqrt{-1}$.

However, when Hamilton attempted to apply the basic rules of arithmetic to quaternions, something quite revolutionary happened. While he could add and subtract quaternions by simply applying these arithmetic operations to the four terms individually, multiplication was much more tricky. He needed a way of multiplying quaternions so that the result is also a quaternion, satisfying the property of closure in group theory. What he found is that this would only be possible if these relationships held:

$$jk = i,\ kj = -i,\ ki = j,\ ik = -j,\ ij = k,\ ji = -k$$

However, these relationships mean that multiplication in quaternion algebra is not commutative. For instance, if $p$ and $q$ are quaternions, $pq$ does not equal $qp$. This was a great shock to mathematicians. For as Kline tells us, "Here was a physically useful algebra which fails to possess a fundamental property of all real and complex numbers, namely that $ab = ba$."[20] This result led mathematicians into quite new forms of algebra, where the objects being operated on are not necessarily numbers, even more revolutionary.

For instance, drawing on Duncan F. Gregory's generalizing principles, Boole helped free mathematics from the tyranny of number systems, regarding the essence of mathematics as "the study of form and structure rather than content, and that 'pure mathematics' is concerned with the laws of combination of 'operators' in their widest sense." As a result, in 1901, Bertrand Russell paid Boole the high compliment of having 'discovered' pure mathematics, although half a century later he said that he had said this to emphasize the central importance of Boolean abstractions.[21] For instance, Boole noted that the commutative and distributive laws of arithmetic could equally apply to differential operators and geometric transformations.[22]

However, the fellows of the Royal Society did not readily accept Boole's major contribution to what is called 'operator theory', for he was precocious autodidact, working outside the constraining mainstream of mathematics at the University of Cambridge. As Gregory advised him, becoming an undergraduate at Cambridge, which Boole made some tentative inquiries to do, would have been unbearable to a man of his intellect and hunger for original research, even if Boole had had the funds to attend the university.[23] For Boole was the son of a shoemaker, who was much more interested in science, literature, and mathematics to attend fully to his business.[24]

So to support his parents and siblings, Boole had been a humble schoolteacher from the age of sixteen, setting up his own school in Lincoln at nineteen.[25] Thankfully, one of the referees—Phillip Kelland, Professor of Mathematics as the University of Edinburgh—saw the merits of Boole's paper and strongly recommended its publication. As a result, Boole was awarded the Royal Society's first gold medal for mathematics, known as the Royal Medal.[26]

By thus gaining a reputation as one of the leading mathematicians of his day, Boole applied for and was appointed the first professor of mathematics at Queen's College in Cork in 1849, even though he did not have a degree.[27] There he met Mary Everest, the niece of John Ryall, the Professor of Greek at the College, and Lieutenant-Colonel Sir George Everest, the Surveyor-General of India, who gave his name to the world's highest mountain.[28] Although Boole was seventeen years older than Mary, the daughter of a clergyman, they married in 1855, having some remarkable progeny.[29] One of the most noteworthy was Alicia Boole Stott, who had the rare ability to visualize four-dimensional space, mentioned on page 129 in Chapter 1, 'Starting Afresh at the Very Beginning'.

Regarding the books with which Boole laid down the initial principles of symbolic logic, he was following a long line of thinkers, from Aristotle to Leibniz, who had dreamt of making logic a precise science, whichpage 91 in Chapter 1, 'Starting Afresh at the Very Beginning' could be formalized and symbolized in such a manner that these principles could be applied "in a more or less mechanical or automatic way to the analysis of a wide range of human, linguistic, ethical, and scientific situations".[30] Of course, such an aim was bound to fail, for it was based on the fourth pillar of unwisdom, on the false belief that human beings are machines and nothing but machines.

Nevertheless, Boole's books were to have an immense influence, building on his earlier methods of abstraction and generality, which have reached their ultimate culmination with the Principle of Unity, the Ultimate Yoga, as we saw on page 91 in Chapter 1, 'Starting Afresh at the Very Beginning'. In *Laws of Thought*, Boole said in his first proposition that all the operations of language, as an instrument of reasoning, could be considered to be a set of literal symbols, such as $x$, $y$, and $z$, representing concepts, and signs of operation, such as +, -, and ×, with a sign of identity =, to compare expressions. For instance, $x$ could denote 'all men', while $y$ could serve as a representation of the class 'good things'. So $xy$ would denote 'all good men', as would $yx$.[31] Alternatively, $y$ could denote 'all women', exclusive to $x$. In this case, $x + y$ would denote 'all men and women'. And $z(x + y) = zx + zy$ could denote 'European men and women', these expressions obeying commutative and distributive laws.[32] It seems, however, that Boole did not consider the associative law, one of the oversights in his trail-blazing work.[33]

Now one key result of his algebraic logic is that if $x$ and $y$ have the same signification, their combination expresses no more than either of the symbols taken alone would do. In this case $xy = x$, as 'good, good things' is essentially the same as 'good things', just with added emphasis. But as $x = y$, we can say $xx = x^2 = x$.[34] By the Principle of Unity, there are two ways that we can view this relationship.

First, in algebra, this equation has the roots 0 and 1, constants that Boole took to denote the empty and universal set or class, respectively. So the expression $1 - x$ would denote 'all things that are not $x$' within some particular domain of discourse. Furthermore, as $x^2 = x$ could be rewritten $x(1 - x) = 0$, this equation represents Aristotle's Law of Contradiction, not as the most basic of axioms, but as a proposition. For as he said, if $x$ represents 'men' and $1 - x$ 'not men', then the expression $x(1 - x)$ represents "*a class whose members are at the same time men and not men,*" which is the empty class, for such a set cannot exist if Aristotle's Law of Contradiction is universally true. Boole called this equation the 'law of duality',[35] later to be called by academics at Cambridge University 'Boole's equation', for in general, it applies no matter what class of beings that $x$ might denote. As Mary his widow tells us, "George afterwards learned, to his great joy, that the same conception of the basis of Logic was held by Leibnitz, the contemporary of Newton."[36]

Alternatively, we could write the equation $x^2 = x$ as an expression or function, $f(x) = x^2 - x$ or $x(x - 1)$, which could take values other than 0. For instance, $x(x - 1) = 1$ could be interpreted as the union of all opposites is the Universe or Wholeness. In other words, the Principle of Unity. As such, Boole's function could represent both the Principle of Unity and the Law of Contradiction, depending on whether it is equal to 1 or 0, respectively.

Boole's function is an example of a general principle by which Boolean algebra could be used for the purely symbolic manipulation of classes. In an example given by Desmond MacHale, much clearer than in Boole's book, consider the classical syllogism 'all As are B, all Bs are C; therefore all As are C'. "In Boole's notation, the hypothesis could be written $a = ab$, $b = bc$. By substitution $a = ab = a(bc) = (ab)c = ac$."[37]

As the result of this seminal book, Boole's name has been immortalized in the operators of AND, OR, and NOT in Boolean algebra, well familiar to anyone engaged in making searches of databases on the Internet, and in the Boolean data type in many programming languages, having the values 'true' or 'false'. Basic arithmetical operations of binary digits can also be represented in Boolean algebra, as the one-bit adder in Figure 8.3 on page 626 shows.

So Boole could be considered one of the founding fathers of computer science, as much as Charles Babbage and Ada Lovelace, as parents who worked on Babbage's Analytical Engine. However, in 1901, Mary Everest Boole, added a postscript on the real meaning of Boole's contribution in a remarkable open letter written to a Dr Bose called 'Indian Thought and Western Science in the Nineteenth Century', published in *The Ceylon National Review* in

June 1909 and printed in booklet form in 1911 under the title of *The Psychologic Aspect of Imperialism*, stretching to twenty-one pages in her voluminous collected works.[38]

Figure 9.1: *Mary Everest Boole*

Mary was a widow for some 52 years, living to the age of 84, to her husband's 49, having had five daughters with him during nine years of marriage. She seems to have been one of the few people who understood the real intention behind his life's work, which is well explained by her letter. Although she was the daughter of a clergyman in the Church of England in Gloucestershire, her father was far from being conventional. Because of suspected consumption, he moved to Paris in 1837, when Mary was five, to be near Samuel Hahnemann, the founder of homoeopathic medicine.[39] On returning to England when Mary was eleven, her father was not a typical priest, regarding himself as a servant of the people, appointed to organize the culture of the parish *in accordance with the desires of the most serious and wise inhabitants*, much to the alarm and anger of the neighbouring clergy.[40]

An incident when preparing for confirmation sheds much light on her religious convictions. Mary asked her father what does it mean to say that Jesus is an Incarnation of God? He replied, "Why can't you understand? You are an Incarnation of God yourself." As she added, "This from a country clergyman in 1849!"[41] Also, she tells us that when her uncle George went out to India at sixteen, "He made the acquaintance of a learned Brahman, who taught him—not the details of his own ritual, as European missionaries do, but—the essential factor in all true religion, the secret of how man may hold communion with the Infinite Unknown."[42] She, herself, was horrified by the British governing classes and colonial attitudes, saying, "how can we expect to retain the loyalty of Hindus, if we trample out their normal development and their self-respect?"[43] And even though she knew that naming the highest mountain in the world after her uncle was to honour his services to engineering science, she still thought that altering the ancient name of the great mountain was a "queer kind of vandalism".[44]

This spiritual, almost mystical background, intuitively grounded in the ancient, perennial wisdom that underlies all the religions, shows us clearly how George and Mary Boole saw *The Laws of Thought*: it was as much a book about psychology as mathematics. For what else is logic, as the science of reason, but the foundation of psychology? To Boole, the human mind works both by receiving information from the external world and also by receiving knowledge directly from The Unseen every time it returns to the thought of Unity between any given elements (of fact or thought), after a period of tension on the contrast or antagonism between those same elements, an insight that arose from his mystical experience as a seventeen-year-old.[45]

However, this was not how the academic world saw this work, even though it was enchanted by it, Herbert Spencer saying that the book was "the greatest advance in Logic since Aristotle". Rather, "nearly all the logicians and mathematicians ignored the statement that the book was meant to throw light *on the nature of the human mind*; and treated [Boole's equation] entirely as a wonderful new method of reducing to logical order masses of evidence about external fact." To which Mary added, "Only think of it! The great English religious mind, which considers itself competent to preach *the Truth*, the only saving Truth, to all mankind; the great academic educational mind which is to improve Hindu culture off the face of the earth, fell into a trap which I believe would hardly have deceived a savage."[46]

As Mary tells us, her husband "went very little into university society, because he had good reason to know that the cordiality of his admirers would in most cases have been diminished if they had had any clear idea what his books really were about."[47] Mary herself had seen since the age of eighteen that 'Boole's equation' is "the mere algebraic expression of natural psycho-logic truth". However, every attempt on her part to explain Boole's equation or function as a law of the human mind known in Asia from the earliest recorded ages met with either violent opposition or blank non-intelligence by her contemporaries.[48] Apparently, few could see what she could see and feel, that the mystical strand underlying the religions

> consists of allusions to and hints of the great, world-wide, world-old secret, of the means by which man can maintain and increase his capacity for directly receiving into himself fresh force from cosmic sources, and fresh knowledge direct from that storehouse of the As-Yet-Unknown which remains always infinite, however much we may learn. I call this latter strand 'secret', not because those who most truly know it are unwilling to communicate it to anyone who wishes to know it, but because of the unwillingness of men agglomerated in groups either to know it or to let it be known. The majority both dislike for themselves the stern self-discipline which the knowledge of it imposes, and dread the mental power given to others by its possession.[49]

Given the resistance Mary felt to George and her attempts to base mathematical logic—the most fundamental of all the sciences, as the science of thought and consciousness—on mystical union with the Divine, which she had learnt from the East, it is perhaps not surprising that she mentions, "I am sometimes told that my experiences and my husband's are unique." To which she responded, "I do not think so. If they were, they would be in no way worth recording." For she suspected that the resolute determination of religious people to suppress evidence of the value of cultures other than their own had led to much work similar to her husband's being ruthlessly destroyed, giving two examples.[50] It seems that nothing has changed in one hundred years. Today, academics in all disciplines, as much as religionists, do their best to deny the universal truth that would enable us to live in love and peace with each other by ending the long-running war between science and religion.

## Mathematical logic

Mathematics, having freed itself from attachment to number and space, then began to become ever-more abstract in the patterns and relationships that mathematicians discovered, whether or not these mapped to physical or metaphysical reality, including mental processes. These are often called 'occult', from Latin *occultus* 'secret', past participle of *occulere* 'to cover over'. In this sense, Isaac Newton—searching for *prisca sapientia* 'ancient wisdom'—was an occultist with his alchemical experiments and theological studies, as we see on page 973 in Chapter 11, 'The Evolution of the Mind'.

In the case of mathematical logic, its evolution is a long, confused story, which is mostly irrelevant to understanding "the fundamental laws of those operations of the mind by which reasoning is performed." Essentially, this is because mathematical logicians developed their subject solely in the horizontal dimension of time, without realizing that they were actually diving into the depths of the Cosmic Psyche, seeking a sound foundation for all knowledge and human learning.

It is not surprising that several of them suffered severe mental disturbances, some spending time in psychiatric institutions, as Gian-Carlo Rota tells us in *Indiscrete Thoughts*. These included Charles Sanders Peirce (1839–1914), Georg Cantor (1845–1918), Giuseppe Peano (1858–1932), Ernst Zermelo (1871–1953), Emil Leon Post (1897–1954), and Kurt Gödel (1906–1978). "Alonzo Church [(1903–1995)] was one of the saner among them, though in some ways his behaviour must be classified as strange, even by mathematicians' standards."[51]

In addition, Gottlob Frege (1848–1925), in later years, at least, was "a man of extreme right-wing political opinions, bitterly opposed to the parliamentary system, democrats, liberals, Catholics, the French and, above all, Jews, who he thought ought to be deprived of political rights and, preferably, expelled from Germany".[52] In contrast, Bertrand Russell (1872–1970) was a peacemaker, being imprisoned for four and a half months in 1918 for writing Pacifist propaganda.[53]

Yet even Russell feared he would go mad, like his uncle William—the son of a British prime minister—who spent the last fifty-eight years of his life in an asylum, a secure place of refuge for those in need.[54] At the time of his death aged ninety-seven in 1970, "Russell left two embittered ex-wives, an estranged schizophrenic son and three granddaughters who felt themselves haunted by the 'ghosts of maniacs', as Russell himself had described his family back in 1893." Five years later, one of these granddaughters committed suicide by setting fire to herself aged twenty-six.[55] And Alan Turing (1912–1954) committed suicide presumably for having been convicted of homosexual acts, which were illegal at the time, even though he was a wartime hero, having deciphered the German Enigma machine.[56]

But concerning the history of ideas, what is particularly interesting in the evolution of mathematical logic is the algebra of relations, which can be traced back to Augustus De Morgan (1806–1871) and Charles S. Peirce, for this evolved into Ted Codd's relational model of data and hence into Integral Relational Logic. Curiously, this pedigree is not mentioned by any of the contributors of essays at the Charles S. Peirce Sesquicentennial International Congress held at Harvard University in 1989,[57] even by Geraldine Brady, who has since done much to put Peirce's logic into its historical context,[58] or by John F. Sowa, who embodied Peirce's meaning triangle in the conceptual structures of artificial intelligence, as we see on page 126 in Chapter 1, 'Starting Afresh at the Very Beginning'.

However, the propositional calculus and first-order predicate logic also played a part in this evolutionary process, so we also need to look at these. However, it is not so easy to trace this process, for many different notations appeared along the way, which can hide the formation of logical concepts, also disguising to what extent pioneers were in touch with themselves and hence Reality. As Western civilization has been moving further and further away from Reality during the past couple of centuries, experts on mathematical logic are not generally very good at throwing light on this psychological process.

In contrast to Boole, De Morgan, nine years older, *did* study mathematics at Cambridge University, graduating as fourth wrangler, that is with first-class honours or *summa cum laude*. However, De Morgan, the son of an army captain of Huguenot descent in the East India Company and the grandson of James Dodson, a mathematician well-known in his day, was not awarded a fellowship because of his unorthodox religious views. Rather, in 1828, at the age of just twenty-two, he was appointed the first professor of mathematics at the new and nondenominational University of London (now University College), the youngest of thirty-two candidates.[59]

Although De Morgan was primarily a brilliant and popular teacher, his "mathematical and literary output was very extensive, probably the largest of any mathematician of his time," that is until Peirce. De Morgan published his first book on *Formal Logic* in 1847, the same year as Boole's *The Mathematical Analysis of Logic* was published. This shows that De Morgan, unlike Boole, was more a reformer of the old logic than a creator of a new one.[60] He began his investigations, primarily into syllogistic reasoning, by saying that Logic is "that part of reasoning which depends upon the manner in which inferences are formed, and the investigation of general maxims and rules for constructing arguments, so that the conclusion may contain no inaccuracy which was not previously asserted in the premises."[61]

So De Morgan studied within the linear framework of Western thought, which led to the invention of the stored-program computer a century later, but which can tell us little about the human mind, consciousness, and hence what the Universe is and how it is designed. Specifically, De Morgan saw logic "as a formal science, having nothing to do, directly, with questions of empirical psychology or abstract metaphysics. Its forms are forms of possible thinking, rather than of actual thought."[62] It was thus that mathematicians began to open up the split between logic and psychology—despite the Booles' worthy intentions—leading scientists today to assert that robots are about to take over the world, making humanity redundant.

De Morgan's *Formal Logic* also contains a chapter on what he called mathematical induction, a term introduced four years earlier in *The Penny Cyclopædia of the Society for the Diffusion of Useful Knowledge*.[63] In 1945, in *How to Solve It*, George Pólya (1887–1985) wrote that it was unfortunate the word *induction* is used in both science and mathematics, "because there is very little logical connection between the two processes."[64] However, De Morgan saw a connection because he first defined induction as "the inference of a universal proposition by the separate inference of all the particulars of which it is composed," which we explore in the next section. He then extended this into mathematics, where the particulars are beyond enumeration, but which nevertheless form a sequence, which can be mapped to the integers.[65] Euclid's proof that there are an infinite number of primes is one of the first uses of mathematical induction as a method of reasoning.[66]

Peirce's view of the relationship between logic, mathematics, psychology, and philosophy is much more complex, apparently varying over the years as his consciousness broadened and deepened, not the least as the result of a mystical experience in 1892.[67] To understand Peirce's ontogeny, we first need to know something of Peirce's family history. His five times great grandfather was John Pers (ca. 1588–1661), a Puritan weaver who moved from Norwich, England in 1637 to settle in Watertown, Massachusetts. So *Peirce* is actually pronounced *purse*. It was Peirce's great grandfather Jerathmiel (1747–1827) who changed the spelling of the family name, moving to Salem and prospering in the East India shipping trade. His son Benjamin (1778–1831) graduated from Harvard College, entered the shipping trade with his father, became a state senator, and, when Salem's shipping trade declined, became Librarian at Harvard, published a four-volume Catalogue of the library's holdings, and wrote a history of the university, which was published shortly after his death.[68]

His son Benjamin (1809–1880), Charles' father, became professor of mathematics and astronomy at Harvard, the leading American mathematician of his day. This Benjamin married Sarah Hunt Mills, the daughter of Elijah Hunt Mills, US senator for Massachusetts, the great great grandfather of Henry Cabot Lodge, Jr (1902–1985), the vice presidential candidate to Richard Nixon in the 1960 presidential election, won by John F. Kennedy. Three of the seven men who founded the National Academy of Sciences with Abraham Lincoln in 1863 were his father, Admiral Charles H. Davis, his mother's brother-in-law, the father-in-law of the elder Henry Cabot Lodge (1850–1924), a leading US senator, who married Peirce's first cousin, and Alexander Dallas Bache (1806–1867), his employer as the Supervisor of the U.S. Coast Survey.[69]

Peirce's first wife Harriet Melusina Fay, affectionately known as Zina, who he married when he was 23 and she 26, had a similar prestigious social background, being the granddaughter of John Henry Hopkins, the first bishop of the Episcopal Diocese of Vermont and the eighth Presiding Bishop of the Episcopal Church in the United States of America. Her father was also an Episcopalian clergyman, also called Charles, who conducted the service. Before the marriage, Peirce was confirmed by his grandfather-in-law-to-be, converting from unitarianism to trinitarianism.[70] We can compare this to Isaac Newton, Fellow of Trinity College, Cambridge, who was a closet unitarian and Arian, denying the divinity of even Jesus, as we see in Chapter 11, 'The Evolution of the Mind' on page 968.

So, on the face of it, Peirce came from an orthodox, upper-middle class family. However, this was far from being the case. On the day of Charles's birth, Benjamin wrote prophetically that he saw his son becoming a genius, celebrity, and great philosopher,[71] rather like the way astrologers have foreseen the destiny of the new-born through the ages. So Charles was the favoured son, even though the second born, destined to be his father's intellectual and spiritual heir.

Most importantly, the son inherited his father's evolutionary cosmology, elucidated in a series of lectures that Benjamin gave in 1879 at the Lowell Institute, published posthumously in 1881 as *Ideality in the Physical Sciences*. As his eldest son, James Mills Peirce, explains in the introduction to this book, Augustus Lowell invited Benjamin "to express his views on the true attitude of science to religion". For this eminent mathematician and astronomer did not see any division between the two. He was quite certain that the physical universe could not have come into being without God the Creator. As he said, "no finite agent can accomplish an infinite production." With the nebular theory of Emanuel Swedenborg (1688–1772), Immanuel Kant (1724–1804), and Pierre-Simon Laplace (1749–1827) as a background,[72] here is Benjamin Peirce's description of his cosmogonic worldview:

> The universe … commences with an all-pervading substance, in which there is no apparent structure nor division into parts, but the same monotonous uniformity throughout. Passing through innumerable transformations, it terminates in a system, whence disorganization has been wholly eliminated, and where vast multitudes of individuals, each a perfect organism in itself, are combined in indestructible harmony. In the beginning, it has the unity of monotony; in the end, it has the unity of complete organization.[73]

Now while this passage refers to the physical universe, it applies equally to the Universe, viewed as Consciousness, essentially a mystical cosmogony, very similar to that described in this book. Most particularly, underlying the Universe is a continuous substrate, out of which all forms evolve and become perfectly organized. However, Benjamin Peirce recognized that his ideality was still evolving as work in progress, for he began the second lecture on 'Cosmogony' with these words:

> How come we here, in the physical world, so curiously adapted to our material and spiritual nourishment? This is one of the first questions proposed by thinking man, as soon as he begins to reason. The inquiry into the origin of the world is, almost instinctively, the beginning of scientific speculation; whereas the complete solution of the question can be achieved only at the very close. So long as a scientific doubt remains, the story of cosmogony is partially untold.[74]

So what Benjamin was not ready to say, like modern evolutionaries, is that the complete organization of the Universe is only reached at the Omega Point of evolution, whence the Universe returns to Ineffable Wholeness through the opposite process of involution.

We can see clearly that Charles was a precocious genius from some biographical notes that he wrote when he was twenty, studying at Harvard. In 1850, when he was ten or eleven, he "Wrote a 'History of Chemistry' " which he began to study when seven years of age, as an antidote to being "seriously and hopelessly in love". He then "Worked at Mathematics for about six months in 1854, "Read Schiller's *Æsthetic Letters*, and began the study of Kant" in 1855, aged about fifteen.[75] He went to Harvard College that same year, graduating with an A.B. before he was twenty, but very low in the class, upgraded to A.M. in 1862. He then graduated *summa cum laude* with a bachelor of science in chemistry at the Lawrence Scientific School in 1863, having also been a regular aide to the U.S. Coast Survey for most of his two and half years' study. As if this was not enough, he and Zina married in the middle of this period, living with his parents at first.

In 1865 he gave a series of eight lectures at Harvard on 'The Logic of Science', given again the following year in a somewhat different form at the Lowell Institute as 'The Logic of Science, or Induction and Hypothesis', in which he planted the seeds for his life's work. Peirce began the first lecture, which effectively launched his career as a logician, although this was not published until 1982, by saying, "The one great source of error in all attempts to make a Logic of Science has been utter misconception of the nature and definition of logic." To try to resolve this confusion, which is still with us today, Peirce said that all the definitions of logic that had evolved during the previous two millennia could be divided into two classes: "those which do not and those which do give to logic a psychological or human character".[76]

In examining the relative merits of these two views of logic, Peirce said, "we ought to adopt a thoroughly unpsychological view of logic", for three reasons. First, "I say that the logical form is already realized in the symbol itself; the psychologists say that it is only realized when the symbol is understood." So "logic needs no distinction between the symbol and the thought; for every thought is a symbol and the laws of logic are true of all symbols." Secondly, Peirce said, "The second advantage of the unpsychological view is that it affords a most convenient means for exploding false notions of the subject," going on to say, "The third advantage of the unpsychological view is that it points to a direct and secure manner of investigating the subject."[77]

Oh dear! In saying this, Peirce was effectively taking meaning out of logic and hence science, a schismatic process that Frege and Russell were to complete at the beginning of the next century, as we see on page 674. By saying that we humans can only think in symbols, he was speaking as a modern computer, which indeed can only think in signs, if machines could think. Concepts, which Peirce later called interpretants, are mental images, which are first formed without words and symbols, which appear secondly, as Einstein wrote in 1945, as we see on pages 126 and 128 in Chapter 1, 'Starting Afresh at the Very Beginning'. This experience conforms entirely with the Peircean cosmogony. But it seems that Peirce junior could not see this at the time.

In saying that Peirce took meaning out of logic, he didn't really, for meaning has never been a part of science, not the least because for thousands of years Western reason has been disobeying the basic law of the Universe: *Wholeness is the union of all opposites*. For in order for our scientific knowledge to be meaningful, it must ultimately be formed and organized within the Cosmic Context of Consciousness or Ultimate Reality. And this is not possible until evolution carries us to its glorious culmination at the end of time.

Nevertheless, while mathematical logic has carried Western civilization into an evolutionary cul-de-sac, it does contain the seeds that we need to become free of this evolutionary dead end. So let us continue our investigations of how we have reached today's parlous situation, because perhaps this could help us extricate ourselves and thereby realize our fullest potential as superintelligent human beings, far surpassing any level of artificial intelligence in machines.

Charles S. Peirce was a major player in this respect, only belatedly being recognized, not the least because like so many creative geniuses, Charley, as he was affectionately known to family and friends, was a troubled soul at times, not easy to live and work with. During childhood, adolescence, and the first half of his career, he had the protection of friends and family, particularly his influential father, who as Supervisor of the Survey from 1867 to 1874 was Charley's boss, seemingly nepotistically promoting him as Assistant to the Supervisor after Bache died. However, things began to go awry in 1876, when Zina left him after fourteen years of marriage.

Amazingly, Zina explained why she had done this in a letter to Carlile P. Patterson, her husband's then boss. She wrote that since childhood, "everything had conspired to spoil him with indulgence," so he did not feel obliged to follow the generally recognized rules of society, including the organization that employed him. So while she recognized his 'brilliant but erratic genius', she nevertheless wrote, "if *only* he will act prudently, cautioning and carefully in everything—instead of rushing things through with recklessness and extravagance—would do him a great deal of good." She concluded the letter with these extraordinary words: "Be good to my Charley, dear Captain Patterson, and be above all judicious with him. Let us save him *together* … if we can."[78]

This letter is a reflection of how difficult it is for a culture that is designed to suppress people's innate intelligence to accommodate those who seek to break free of these shackles. However, this is not just a human problem. It is an example of how any structure—whether physical or psychological—seeks to protect itself in what systems theorists call homeostasis 'same state', a critical situation at these times of unprecedented evolutionary change, as we look at further in Chapter 13, 'The Prospects for Humanity' on page 1027.

But let us look briefly at Peirce's contribution to the foundations of mathematical logic. To keep this simple, I feel that it is easiest to look at a few key elements of mathematical logic as they are today and to trace their origins back to Peirce and others. For to interpret the thoughts behind the original writings, we need to penetrate deeply into the nineteenth and twentieth century mindset, as it existed prior to the invention of the stored-program computer, quite an onerous task.

Peirce's simplest paper on mathematical logic is one he wrote in 1880 titled 'A Boolian [sic] Algebra with One Constant', but not published until 1933 in Volume IV of his *Collected Papers*, titled *The Simplest Mathematics*. As the editors Charles Hartshorne and Paul Weiss pointed out, this unpublished manuscript anticipated Sheffer's landmark paper of 1913,[79] mentioned on page 625 in Chapter 8, 'Limits of Technology'. Then in 1892, Peirce further demonstrated the functional completeness of the NAND and NOR operators, the former known today as the Sheffer stroke, using the term *ampheck*, from Greek *amphēkēs* 'cutting both ways', for the NOR operator.[80] We thus see that Peirce was many years ahead of his time, but not generally recognized until a century later.

At the time, Peirce was working intensively on an uncompleted book titled *Minute Logic*, one of several unsuccessful attempts to write and publish his magnum opus. Chapter 3 of this book is titled 'The Simplest Mathematics',[81] well illustrating Peirce's thoroughness in seeking simple principles on which to build the entire world of learning. He began this chapter with this definition of mathematics taken from his father's 1870 paper on 'Linear Associative Algebra': mathematics is 'the science which draws necessary conclusions'. This definition well illustrates the linear—that is, mechanical—nature of mathematical proof.[82] However, it ignores the fact that the subject of mathematics—when viewed as a whole—is nonlinear, a perspective that Peirce also commented on, as I have read somewhere. It is absolutely essential to make this distinction if we are to discover what it truly means to be a human being, in contrast to machines, like computers.

The most significant point about this chapter is that Peirce carefully examined all possible ways in which a pair of mathematical values, which he called **v** and **f**, for *verity* and *falsity*, could be combined.[83] Curiously, the signs that Peirce used for the sixteen binary operators were omitted from the *Collected Papers* of 1933, as Glenn Clark pointed out in his contribution to the Sesquicentennial International Congress in 1989.[84] In the event, these were not published until 1976 in Volume 3 of *The New Elements of Mathematics*, edited by Carolyn Eisele.[85] There is no need to describe these strange signs here, for they do not really contribute to understanding the underlying concepts. It is simpler to list their modern equivalents in the propositional or sentential calculus in tabular form in Table 9.5.

| **Class name** | *Binary logical operators* | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Attribute name** | | *Symbol* | *Operation* | *Operator* | *Values* | | | |
| **Initial attribute values** | p | | | | T | T | F | F |
| | q | | | | T | F | T | F |
| **Derived attribute values** | 0 | XAND | Contradiction | false | F | F | F | F |
| | 1 | NOR | Logical NOR | $\downarrow$ | F | F | F | T |
| | 2 | | Converse nonimplication | $\not\subset$ | F | F | T | F |
| | 3 | ¬p | Negation | | F | F | T | T |
| | 4 | | Nonimplication | $\nrightarrow$ | F | T | F | F |
| | 5 | ¬q | Negation | | F | T | F | T |
| | 6 | XOR | Exclusive disjunction | $\oplus$ | F | T | T | F |
| | 7 | NAND | Logical NAND | $\uparrow$ | F | T | T | T |
| | 8 | AND | Conjunction | $\wedge$ | T | F | F | F |
| | 9 | XNOR | Logical biconditional | $\leftrightarrow$ | T | F | F | T |
| | 10 | q | Projection function | | T | F | T | F |
| | 11 | | Implication | $\rightarrow$ | T | F | T | T |
| | 12 | p | Projection function | | T | T | F | F |
| | 13 | | Converse implication | $\leftarrow$ | T | T | F | T |
| | 14 | OR | Disjunction | $\vee$ | T | T | T | F |
| | 15 | XNAND | Tautology | true | T | T | T | T |

Table 9.5: *Truth table of binary logical operators as an IRL relation*

Once again, we see here the sleepwalking characteristics of the evolution of human learning. It was not until 1920 that Emil Leon Post (1897–1954) introduced the notion of truth table to display the truth values of binary operators. He did this in his doctoral dissertation at Columbia University, published the following year.[86] Also in 1921, Ludwig Wittgenstein (1889–1951) displayed such a truth table in the *Tractatus Logico-Philosophicus*,[87] apparently discovered independently. Where Peirce had used **v** and **f**, Post and Wittgenstein used + and – and T and F, respectively. Today, truth tables are a powerful way of proving the relationships between the various binary operators.

The propositional calculus is the simplest form of mathematical logic, for rather than looking at individual terms in propositions, as in the syllogism, entire propositions are denoted by a single sign, such as *P*, without any consideration for the structure of the proposition.

Now, having realized that human reason could be represented in mathematical language, to make logic a rigorous discipline, following Euclid, mathematicians also needed to define basic axioms, as assumed truths, and the rules for transforming these axioms into theorems.

Two rules of inference in propositional logic and boolean algebra have been called De Morgan's laws since 1918, after Augustus De Morgan, even though they were known to logicians in the Middle Ages and it is not clear when and where De Morgan defined these rules. These are a pair of transformation rules that allow the expression of conjunctions and disjunctions purely in terms of each other via negation.[88]

The rules can be expressed in English as:

- The negation of a conjunction is the disjunction of the negations.
- The negation of a disjunction is the conjunction of the negations.

The rules can be expressed in formal language with two propositions *P* and *Q* as:

$$\neg(P \wedge Q) \Leftrightarrow (\neg P) \vee (\neg Q)$$

$$\neg(P \vee Q) \Leftrightarrow (\neg P) \wedge (\neg Q)$$

where $\Leftrightarrow$ is a metalogical symbol meaning 'can be replaced in a logical proof with'.

Another mechanism for the construction of deductive proofs, known since antiquity, is *modus ponens* or the rule of detachment, which states that if *P* is true and if *P* implies *Q*, then *Q* must be true. In formal sequent notion, *modus ponens* is defined thus:[89]

$$P \rightarrow Q, P \vdash Q$$

where $\vdash$ is a metalogical symbol meaning that *Q* is a syntactic consequence of $P \rightarrow Q$ and *P* in some logical system. It is also possible to express the rule of inference as a theorem in tautological form, showing that it is possible to represent *modus ponens* rather like passive and active data in computers, defined on page 625 in Chapter 8, 'Limits of Technology':

$$((P \rightarrow Q) \wedge P) \rightarrow Q$$

This brings us to the strange case of the axioms of propositional logic. They are all tautologies, true no matter what the values of individual propositions, such as *P* and *Q*, might be. As such, they do nothing to lead us to the Truth. I suspect that this is why I lost interest in mathematics as an undergraduate in the early 1960s, although I was too depressed to be able to articulate my thoughts and feelings in the way I can do fifty years later. As I can see now, I must have realized that the whole of Western thought, including my beloved mathematics, was completely meaningless. For further reference, Figure 9.2 shows a set of four axioms for propositional logic, essentially those of *Principia Mathematica*:[90]

Ernest Nagel and James R. Newman well illustrate the meaninglessness of tautologies with interpretations of these axioms. For there is no need in the propositional calculus for propositions to have any semantic relationship to each other. For instance, Table 9.5 is the example they use for the fourth axiom:[91]

$$(P \vee P) \to P$$

$$P \to (P \vee Q)$$

$$(P \vee Q) \to (Q \vee P)$$

$$(P \to Q) \to ((R \vee P) \to (R \vee Q))$$

Figure 9.2: *Axioms for propositional calculus*

| | |
|---|---|
| $P$ | Ducks waddle |
| $Q$ | 5 is a prime |
| $R$ | Churchill drinks brandy |

Table 9.6: *An instance of propositional variables*

This axiom reads: If (if ducks waddle then 5 is a prime) then (if (either Churchill drinks brandy or ducks waddle) then (either Churchill drinks brandy or 5 is a prime)). Now, who on earth would ever say such a sentence? There is an excellent example of the great gulf between mathematical logic and cognitive psychology. In this example, all the propositions are generally regarded to be true. But this is not necessary; any one of them could also be false, illustrated in Table 9.7, the truth table for the third axiom.

| $P$ | $Q$ | $P \vee Q$ | $Q \vee P$ | $(P \vee Q) \to (Q \vee P)$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | T | T | T |
| F | T | T | T | T |
| F | F | F | F | T |

Table 9.7: *Truth table for a tautology*

Needless to say, the propositional calculus played very little part in the development of Integral Relational Logic. In the spring of 1980, I did spend a couple of weeks playing with truth tables, but this led nowhere. It was not until midsummer that year that the Principle of Duality emerged in consciousness, from the principle of duality in projective geometry—as described in Section 'The Principle of Duality' in Chapter 3, 'Unifying Opposites' on page 225—that my life took off. I haven't looked back since.

However, some other features of mathematical logic did play a role in the formulation of Ted Codd's relational model and hence in Integral Relational Logic. The most important of these is the logic of relations, for relation is a primal concept in IRL. As far as I can tell, Augustus De Morgan was the first to write about the logic of relations, in the fourth of five papers he wrote 'On the Syllogism' for the *Transactions of the Cambridge Philosophical Society*, between 1846 and 1862, However, he pointed out in the introduction to this fourth paper, dated 12th November 1859, but read on 23rd April 1860, that he had already mentioned relations in the second and third papers dated 1850 and 1858, for "the ordinary syllogism [is] one cane, and one case only, of the *composition of relations*."[92]

De Morgan first defined *relation* in his 1858 paper thus: "When two objects, qualities, classes, or attributes, viewed together by the mind, are seen under some connexion, that connexion is called a relation."[93] He thus generalized the notion of the copula, from Latin *cōpula* 'link', which connects the subject and predicate in syllogistic propositions. For as Morris Kline points out, the relation 'to be' is severely limited, leading to incorrect or possibly incorrect conclusions. He gives two examples:[94]

John is a brother;
Peter is a brother;
Hence John and Peter are brothers (of each other),

which can obviously be incorrect. Likewise:

An apple is sour;
Sour is a taste;
Hence an apple is a taste,

is also an incorrect conclusion. As De Morgan realized, the richness of the relationships between thoughts, although much studied by psychologists to his time, had been neglected by logicians. He mentions that Aristotle had paid scant regard to the abstract notion of relation, although in so doing, "Aristotle is rather too much the expositor of common language, too little the expositor of common thought."[95] In extending Boole's mathematical laws of thought, it seems that De Morgan was attempting to go beyond the structure of sentences and look at the underlying structure of thought, and hence of the Universe. However, he was still stuck with signs or symbols for thoughts, not able to see the data patterns that exist prior to interpretation as mental images, which, in themselves, have no sign, as we see in Section 'Concept of concept' in Chapter 1, 'Starting Afresh at the Very Beginning' on page 111.

Charles S. Peirce, a voracious reader of everything he could get his hands on (in 1896 he had twenty-nine volumes of the *Philosophical Transactions of the Royal Society* in his library),[96] read De Morgan's 'Logic of Relations' and used it as the basis of a paper he presented on 26th January 1870 to the American Academy of Arts and Sciences, titled 'Description of a Notation for the Logic of Relatives, Resulting from an Amplification of the Conceptions of Boole's Calculus of Logic' (DNLR).[97] This was then published in the *Memoirs of the American Academy of Arts and Sciences* and also as a book, the first of Peirce's published papers in logic.[98] In 1984, Daniel D. Merrill described this paper as "one of the most important works in the history of modern logic, for it is the first attempt to expand Boole's algebra of logic to include the logic of relations".[99]

However, this seminal paper was not well known, for as Geraldine Brady tells us, "The European mathematical and scientific community would have had little contemporary access to Peirce's paper except through personally circulated copies."[100] Indeed, this is how some leading logicians, such as De Morgan and W. Stanley Jevons, first came to know of Peirce's work. In 1870, Peirce visited Europe for the first time in his capacity as assistant to the US Coast Survey to find a suitable site to watch the eclipse of the sun in December that year. So in June, he was able to meet De Morgan, giving him a copy of DNLR, which was discussed at the Liverpool meeting of the British Association for the Advancement of Science in September.[101]

During the 1870s, Peirce continued to develop his ideas on the algebra of logic, even when working for the US Coast and Geodetic Survey, as it became in 1878,[102] suffering three nervous breakdowns between 1875 and 1877, no doubt partially brought on my overwork, suffering six others in 1879, 1884, 1904, 1905, 1909, and 1911.[103]

Towards the end of 1877, Benjamin Peirce, being concerned for his son's health and salary—limited by Congressional budgets—wrote to Daniel Coit Gilman, President of the newly formed Johns Hopkins University in Baltimore, recommending Charles as head of the department of physics. However, Peirce junior regarded himself more as a logician than a physicist, so wrote to President Gilman in January 1878, asking for the opportunity to develop his sketchy ideas on mathematical logic at what Peirce called "the only real university in America". However, he also wrote that he could not give up his pendulum research at the Coast Survey; he wished to be paid for two jobs, both of which were full-time.[104]

In the event, it took eighteen months for a satisfactory arrangement to be made, Peirce being appointed as a lecturer in logic on 6th June 1879, not as a professor, as he would have liked.[105] As a result, he wrote a major paper on 'On the Algebra of Logic', published in 1880 in the *American Journal of Mathematics*, founded and edited by James Joseph Sylvester (1814–1897), professor of mathematics at Johns Hopkins University.[106] This appointment was renewed annually for the next few years, leading to the publication in 1883 of *Studies in Logic*, by some of Peirce's student, most notably O. H. Mitchell and Christine Ladd, with an important appendix by Peirce on 'The Logic of Relatives'.[107]

As a follow-on to the 1880 paper, Peirce read a paper in October 1884 before the National Academy of Sciences, published in January 1885 in expanded form as 'On the Algebra of Logic: A Contribution to the Philosophy of Notation' in the *American Journal of Mathematics*, intended as the first of two papers for this journal.[108] In the event, this "was to be Peirce's last technical paper on logic to appear in a major scientific journal",[109] although he did have an article published on 'The Logic of Relatives' in *The Monist* in 1897.[110]

The primary reason for Peirce's change of fortunes was his nemesis, Simon Newcomb (1835–1909), a quite different character from Charles S. Peirce, as you can see from the photograph in Figure 9.3. Newcomb, a protégé and friend of Benjamin Peirce, succeeded Sylvester as the editor of the *American Journal of Mathematics* in 1885 and refused to publish the second part of the 1885 paper, "on the ground that its subject was not mathematics".[111]

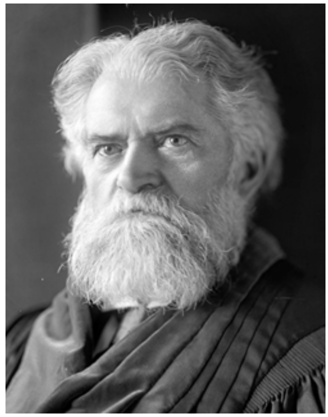


Figure 9.3: *Simon Newcomb*

Without going too deeply into Newcomb's psychology, there seem to be three major influences on his attitude to Peirce. First, not coming from Peirce's privileged background, he resented the advantages that Peirce had been given. Secondly, he was probably subconsciously envious of Peirce's brilliant genius. Thirdly, and most importantly, he was appalled by the way that Peirce married his mistress Juliette, having lived openly with her after he and Zina separated. For as Joseph Brent writes, "For a sanctimonious man of affairs of the period such as Newcomb, for Peirce to have a mistress was both understandable and acceptable if the affair were carried on discretely, but to marry her after such a public liaison was outrageous because to do so attacked the sanctity of marriage."[112]

Newcomb's hostility was to lead Peirce into dire financial straits during the last twenty-two years of his life, not even receiving a pension from his thirty-one years with US Coast and Geodetic Survey. First, Newcomb was instrumental in Peirce's effective dismissal as a lecturer at John Hopkins University in 1884, rather than being offered tenure, as President Gilman had previously seemed to be disposed to do.[113] Secondly, in 1890, Newcomb was asked to review a report that Peirce had written for the US Coast and Geodetic Survey.[114] While two of the three reviewers recommended it to be published, Newcomb rejected this proposal, which was a major contribution to Peirce being dismissed from the Survey in December 1891.[115] Thirdly, when Peirce applied to the Carnegie Institution for a grant to publish his magnum opus in thirty-six chapters, this was rejected on Newcomb's recommendation in March 1903.[116]

So from 1892 onwards, Peirce did not have a regular source of income, being unacceptable to American Academe, despite being the most original philosopher in the history of the United States. At times in the mid 1990s, "he was so poor that he did not eat for days and had no place to sleep, spending days and nights wandering in [New York] city" with the down-and-outs.[117] From time to time, his friends gave him some money, which his closest friend William James (1842–1910) formalized in 1907 by arranging for between fifteen and twenty-five subscribers to donate to a fund raising about $1000 annually.[118]

During this period of near destitution, Peirce did, however, have a few patrons, who gave him work. One was Paul Carus (1852–1919), the editor of the *The Monist* and *Open Court*, founded by his father-in-law, Edward C. Hegeler (1835–1910), who played "the role of Alexander to Peirce's Aristotle," until they fell out.[119] Peirce was to write a number of significant articles on metaphysics and the relationship of religion and science for these journals.

It is easy to see why Carus was sympathetic to Peirce's philosophy, for he "was a follower of Benedictus de Spinoza; he was of the opinion that Western thought had fallen into error early in its development in accepting the distinctions between body and mind and the material and the spiritual." In pursuit of a religion of science, he was a key figure in the introduction of Buddhism to the West, and after a battle for survival, he expected a 'cosmic religion of universal truth' to emerge from the ashes of traditional beliefs, a vision that is about to be realized.[120] I wonder if knew Richard Maurice Bucke (1837–1902), author of *Cosmic Consciusness*, published in 1901, who had a similar vision, many years ahead of his time.

Another patron was Wendell Phillips Garrison (1840–1907), the editor of *The Nation*, who gave Peirce some 230 books to review, with a remarkable breadth of subject matter, from physics and logic to wine and fine food.[121] However, all attempts to find a publisher in the 1990s to publish his *Grand Logic*, later called *How to Reason: A Critick of Arguments*, failed, and this book was left unfinished, along with some other projects.[122]

William James—Peirce's most ardent supporter, although they did not always see eye to eye—was constantly attempting to use his influence to help Peirce along, not always successfully. For instance, in 1898, James arranged for Peirce to give a series of eight lectures, which Peirce originally proposed *On the Logic of Events*. However, James thought these would be too technical for the audience he had in mind, writing to Peirce in December 1897: "I am sorry you are sticking so to formal logic. … Now be a good boy and think a more popular plan out. … You are so teeming with ideas—and the lectures need not by any means form a continuous whole. Separate topics of a vitally important character would do perfectly well."[123]

This seems to have thrown Peirce into some confusion, as he said in the first revised lecture on 'Philosophy and the Conduct of Life', "just as I was finishing one lecture [on Objective Logic] word came that you would expect to be addressed on Topics of Vital Importance, and that it would be as well to make the lectures detached,"[124] changing the title of the lectures to *Detached Ideas on Vitally Important Topics*. In the event, the lectures were titled *Reasoning and the Logic of Things*, given in a private house in Cambridge, for Harvard University would not allow Peirce on the premises, still regarding him as a *persona non grata*.

These lectures were not published at the time, Peirce's manuscripts containing drafts of both the original and given lectures, which the editors of the *Collected Works* in the 1930s were unable to unravel, as Arthur W. Bucks pointed out in 1958 in Volume VIII of the *Collected Works*.[125] In the event, these lectures were not published until 1992 in as close as a way as possible to what was thought to have been actually given. In their introduction to these lectures, Kenneth Laine Ketner and Hilary Putnam said that *The Cambridge Conferences Lectures of 1898* would have been better titled *The Consequences of Mathematics*, for these lectures provide “an admirable popular introduction to Peirce’s … application of mathematics to philosophy”.[126]

In the exordium for the third lecture titled ‘The Logic of Relatives’, Peirce reiterated his determination to keep logic separate from psychology, saying, “My proposition is that logic, in the strict sense of the term, has nothing to do with how you think.”[127] So even though Peirce’s mystical experience of 1892—described on page 121 in Chapter 1, ‘Starting Afresh at the Very Beginning’—informed much of his philosophy, he was not able let go of his belief that mathematics is the fundamental science on which all others are built. Even his notion of *synechism* ‘continuity’, which James asked him to speak about, along with *tychism* ‘chance’,[128] is based on the mathematical notion of the continuum of the infinitessimals, introduced by Georg Cantor. So even though he sought to be free of a mechanistic worldview, his materialistic conditioning, along with his father’s, inhibited him from discovering that psychology is the primary science, and that our thoughts, along with the entire world of form, emerge from the Formless Absolute.

On this point, it is instructive to note that *exordium* is cognate with *primordial* ‘first in sequence of time, original; primary, fundamental’, both words deriving from Latin *ōrdīrī* ‘to begin to weave’, from PIE base **ar-* ‘to fit together’, also root of *harmony* and *order*, with a similar root sense to *Tantra*. So the ancients in both West and East were clearly aware of the Principle of Unity, the fundamental design principle of the Universe, weaving opposites together into a coherent whole. But the Western mind, especially the mathematical one, has great difficulty in assimilating this fundamental principle in consciousness. For instance, on a number of occasions, Peirce emphasized that his view of logic held on to the absolute truth of Aristotle’s Laws of Contradiction and Excluded Middle, not unifying opposites in what Heraclitus called the ‘Hidden Harmony’.

Given the turmoil in Peirce’s life, it is not surprising that his contribution to mathematical logic has been greatly underestimated. As part of a Ph. D. thesis, in the 1990s, Geraldine Brady did some sterling work on Peirce’s place in the history of logic, particularly his influence on Ernst Schröder (1841–1902), Leopold Löwenheim (1878–1957), and Thoralf Skolem (1887–1963). As she tells us, Peirce’s principal contributions include:[129]

- The calculus of relations
- A lattice-theoretic formulation of Boolean algebra
- Implicative propositional logic
- Quantified propositional logic and Boolean algebra
- Existential graphs
- An axiomatic arithmetic of the natural numbers

There is no need to study these ideas in detail, for they have had very little influence on the evolution of Integral Relational Logic, which is based on a complete break with the past, as described in Chapter 1, 'Starting Afresh at the Very Beginning' on page 35. To the extent that they did influence the development of IRL, the history of mathematical logic can best be seen like the development of an old-fashioned chemical photograph, which is fuzzy at first, eventually reaching clarity when it is fully developed at the Omega Point of evolution at the end of time.

By far the most important aspect of traditional logic is the calculus of relations, for relationships play the central role in the underlying structure of the Universe. The essence of this subject is incredibly simple, far simpler than the mathematicians make it. To illustrate the simplicity of relations, let us take the propositional formula '*X* says that *Y* loves *Z*', where there is a binary relationship between *Y* and *Z*, which we could call *R*, and one between *X* and *R*, a slight modification of an example in Wikipedia.[130] Giving values for *X*, *Y*, and *Z* gives a **relation** in tabular form in Table 9.8 a primal concept in IRL.

| Person X | Person Y | Person Z |
|---|---|---|
| Alice | Bob | Denise |
| Charles | Alice | Bob |
| Charles | Charles | Alice |
| Denise | Denise | Denise |

Table 9.8: *A ternary relation*

However, mathematicians then took over the concept of *relation*, almost completely detaching it from its psychological and linguistic origins. For instance, Wolfram *MathWorld* defines a relation as "any subset of a Cartesian product. For instance, a subset of $A \times B$, called a 'binary relation from *A* to *B*', is a collection of ordered pairs $(a, b)$ with first components from *A* and second components from *B*."[131] In turn, "The Cartesian product of two sets *A* and *B* (also called the product set, set direct product, or cross product) is defined to be the set of all points $(a, b)$ where $a \in A$ and $b \in B$. It is denoted $A \times B$, and is called the Cartesian product since it originated in Descartes' formulation of analytic geometry."[132]

"An illustrative example [taken from Wikipedia] is the Standard 52-card deck. The standard playing card ranks {Ace, King, Queen, Jack, 10, 9, 8, 7, 6, 5, 4, 3, 2} form a 13 element-set. The card suits {♠, ♥, ♦, ♣} form a 4-element set. The Cartesian product of these sets returns a 52-element set consisting of 52 ordered pairs which correspond to all 52 possible playing cards. **ranks** × **suits** returns a set of the form {(Ace, ♠), (King, ♠), …, (2, ♠), (Ace, ♥), …, (3,♣), (2, ♣)}. **suits** × **ranks** returns a set of the form {(♠, Ace), (♠, King), …, (♠, 2 ), (♥, Ace), …, (♣, 3), (♣, 2)}."[133]

Now, the binary relation is just a special case of an *n*-ary relation or set of tuples, where a tuple is an ordered list of *n* elements $(a_1, a_2, a_3, \ldots a_n)$, distinguished from a set, in which the elements are unordered, unique, and potentially infinite. In general, therefore, a relation in mathematics is a subset of the Cartesian product $A_1 \times A_2 \times A_3 \times \ldots \times A_n$. These are the basic concepts on which Ted Codd based his own definition of relation in the relational model of data, given on page 601 in Chapter 7, 'The Growth of Structure'. However, there is some confusion here in the language. Tuples contain meaningful relationships within them, distinct from relations. So in IRL, we distinguish *relations*, as tables, and *relationships* between the primal concepts of **class**, **entity**, and **attribute**.

As well as defining the concept of relation in the relational model, Codd also introduced some basic operations on these relations in what is now known as relational algebra,[134] distinct from relation algebra, introduced by De Morgan and Peirce.[135] Codd introduced these operations because, while relations are sets, not all operations on relations so viewed are relations. For instance, as Codd pointed out, the union of a binary relation and a ternary relation is not a relation. Hence, "These operations are introduced because of their key role in deriving relations from other relations,"[136] leading to the closure of relational algebra.

When I heard Codd speak about these operations at a one-day conference in London in the spring of 1973, when working in an IBM sales office as a senior systems engineer, it blew my mind. For, although I did not fully understand what he was saying, the potential to use mathematical operators to combine and extract information from databases was clearly the right direction in which the data-processing industry should go, giving it what I thought was a solid theoretical foundation. One example of such a relational operator is called the natural join (⋈), of which the Cartesian product is a degenerate case. The natural join combines information contained in two relations into a third, using equal attribute values in columns in the two relations that have the same attribute name and domain of values. For instance, if $R$ and $S$ are Employee and Department relations, given in Tables 9.9 and 9.10, then $R \bowtie S$ is given in Table 9.11.[137]

| Name | EmpId | DeptName |
|---|---|---|
| Harry | 3415 | Finance |
| Sally | 3415 | Sales |
| George | 3401 | Finance |
| Harriet | 2202 | Sales |

Table 9.9: *Employee relation*

| DeptName | Manager |
|---|---|
| Finance | George |
| Sales | Harriet |
| Production | Charles |

Table 9.10: *Department relation*

| Name | EmpId | DeptName | Manager |
|---|---|---|---|
| Harry | 3415 | Finance | George |
| Sally | 3415 | Sales | Harriet |
| George | 3401 | Finance | George |
| Harriet | 2202 | Sales | Harriet |

Table 9.11: *Join of Employee and Department relations*

We do not need to dwell on relational algebra any further, not the least because it has little use in the way we humans arrange our ideas. Even Structured Query Language (SQL), the *de facto* standard programming language for managing data in relational database management systems (RDBMS), is only loosely based on relational algebra and something called 'tuple relational calculus'. On this point, in 1991 and 1992, I worked on a product called LanguageAccess at the IBM Nordic Software Development Laboratory in Stockholm whose purpose was to translate natural language questions made to a relational database into SQL. LanguageAccess managed simple questions quite well, but had great difficulty with more complex ones, so development and marketing was dropped.

The only remaining point that needs to be made is that relational algebra is a subset of first-order predicate logic, from which it has also evolved.[138] This brings us to Gottlob Frege's seminal work *Begriffsschrift*, published in 1879, "generally considered the work that marks the birth of modern logic", as Wikipedia says without any citations.[139] However, as Geraldine Brady tells us, there is a common misconception here. For while Frege "has undisputed priority for the discovery and formulation of first-order logic", this does not mean that his influence was immediately felt by his successors. For while Bertrand Russell was clearly influenced by Frege, largely ignoring Peirce, "the central ideas of what we now call first-order logic were fully implicit in the works of Schröder and Peirce from which Löwenheim drew his chief inspiration."[140]

Furthermore, Frege's notation is so obscure, that no one since has attempted to develop it. In the event, the current notation for first-order logic does not come from Peirce, Frege, Schröder, Whitehead, Russell, or any other such pioneer. "It arrives full blown in Hilbert's 1917 lectures, without any reference to anyone."[141]

Nevertheless, Jean van Heijenoort considers *Begriffsschrift* as "perhaps the most important single work ever written in logic. For its fundamental contributions, among lesser points, are":[142]

- The truth-functional propositional calculus
- The analysis of the proposition into function and argument(s) instead of subject and predicate
- The theory of quantification
- A system of logic in which derivations are carried out exclusively according to the form of the expressions
- A logical definition of the notion of mathematical sequence

So what is *Begriffsschrift*? Well, this is usually translated as 'concept writing' or 'concept notation'. However, like Boole before him, Frege saw this endeavour as an attempt to symbolize the way that we human beings think, as the full title of this short book in English translation indicates: *A Formula Language, Modelled on that of Arithmetic, of Pure Thought*. Also, Philip Jourdain translated *Begriffsschrift* as 'ideograph' in a 1912 paper, a translation that apparently Frege approved.Be this as it may, *Begriff* derives from German *begreifen* 'to comprehend', from the PIE base **ghreib* 'to grip', also the root of *grip* 'grasp, clutch', with a figurative meaning 'Intellectual or mental hold; power to apprehend or master a subject'. So a concept is something that can be held in the mind.

Now Frege was not simply representing Aristotle's logic in symbolic form, as Boole had done; he was seeking to employ logic in order to provide a sound foundation for arithmetic. As he said, "My intention was not to represent an abstract logic in formulas, but to express a content through written signs in a more precise and clear way than it is possible to do through words. In fact, what I wanted to create was not a mere *calculus ratiocinator* but a *lingua characterica* in Leibniz's sense."[143] For Leibniz had distinguished two components in his ambitious project to create a mathematical logic. As Jaakko Hintikka tells us:

> On the one hand, Leibniz proposed to develop a *characteristic universalis* or *lingua characteristica* which was to be a universal language of human thought whose symbolic structure would reflect directly the structure of the world of our concepts. On the other hand, Leibniz's ambition included the creation of a *calculus ratiocinator* which was conceived of by him as a method of symbolic calculation which would mirror the processes of human reasoning.[144]

Frege favoured the former approach, replacing Aristotle's subject and predicate with the mathematical concepts of function and argument, introducing quantifiers and propositional functions into logic.[145] As we have seen in this subsection, these eventually evolved into Integral Relational Logic, thereby fulfilling Leibniz's great dream. However, Leibniz's conception of a *calculus ratiocinator*, explored by Vernon Platt in Part I of *Thinking Machines: The Evolution of Artificial Intelligence*,[146] was bound to fail, for no mechanistic, linear process of reasoning can possibly provide us with a valid map of a nonlinear, holographic, multidimensional Universe, being constantly refreshed through the Divine power of Life arising directly from the Fountainhead.

## Crisis in the foundations

Now Frege was not only interested in developing a language in which to express a science of pure thought, he also sought to provide arithmetic with a sound foundation through his logic, publishing the first volume of *Grundgesetze der Arithmetik* '*Basic Laws of Arithmetic*' in 1893. In this respect, Frege differed markedly from Peirce. While the latter sought to base logic and indeed all philosophy on mathematics, just as science was so based in his time, Frege sought to base mathematics on logic. Of course, this makes much more sense, for psychology, as the science of mind, thought, and consciousness, must be the primary science, underlying all others.

However, this is not how Frege saw the relationship of logic to psychology, for like Peirce, Frege sought to separate logic, as the science of mind and reason, from psychology. Bertrand Russell agreed with them, both writing to Frege on 16th June 1902:

> For a year and half I have been acquainted with your *Grundgesetze der Arithmetik*, but it is only now that I have been able to find the time for the thorough study I intended to make of your work. I find myself in complete agreement with you in all essentials, particularly when you reject any psychological element [*Moment*] in logic and when you place a high value on ideography [*Begriffsschrift*] for the foundation of mathematics and formal logic, which, incidentally, can hardly be distinguished.[147]

However, Russell also pointed out there was a logical flaw in Frege's reasoning because of the paradoxes that he had found in the concept of 'all classes'. Russell was amazed at Frege's humble reply six days later. In giving permission for his correspondence with Frege to be published, Russell said this about his colleague, who he never actually met: "when upon finding that his fundamental assumption was in error, he responded with intellectual pleasure clearly submerging any feelings of personal disappointment. It was almost superhuman and a telling indication of that of which men are capable if their dedication is to creative work and knowledge instead of cruder efforts to dominate and be known."[148] For Frege wrote to Russell:

> Your discovery of the contradiction caused me the greatest surprise and, I would almost say, consternation, since it has shaken the basis on which I intended to build arithmetic. … It is all the more serious since, with the loss of my Rule V, not only the foundations of my arithmetic, but also the sole possible foundations of arithmetic, seem to vanish. … In any case your discovery is very remarkable and will perhaps result in a great advance in logic, unwelcome as it may seem at first glance.[149]

In the event, Frege did publish the second volume of *Grundgesetze der Arithmetik* in 1903, with an appendix on Russell's paradox, and Russell published *Principles of Mathematics* the same year, with two appendices, titled 'The Arithmetical and Logical Doctrines of Frege' and 'The Doctrine of Types'. In this second appendix, Russell proposed a tentative solution to paradoxes, which he called the theory of types. In this, he distinguished terms and individuals from their ranges of significance, determined, for instance, when grouped in classes.[150]

The problem of formalizing human reason arose because paradoxes were found in set theory, as shown page 236 in Chapter 3, 'Unifying Opposites'. So how could mathematicians recover from this critical situation? Well, at the International Congress of Mathematicians in Paris in 1900, David Hilbert (1862–1943), being deeply concerned about the state of mathematics at the turn of the century, presented twenty-three unsolved problems in mathematics.[151] The second of these was concerned with proving that the axioms of mathematics are both independent and consistent.[152] As Hilbert put it with regard to the axioms of arithmetic, he asked mathematicians "To prove that they are not contradictory, that is, that a definite [finite] number of logical steps based upon them can never lead to contradictory results."[153]

We can well demonstrate that the Western mind's aversion to paradoxes and self-contradictions is deeply embedded in the cultural psyche by A. N. Whitehead and Bertrand Russell's *Principia Mathematica*, an initial response to one of Hilbert's challenges. In their futile search for certainty in mathematics and science, these fellows of the Royal Society wrote this monumental treatise in the second decade of the last century in order to deny the basic principle on which the Universe is designed. They took 360 pages to prove the proposition (*54.43) that would eventually lead to the arithmetical statement '1 + 1 = 2',[154] including several incomprehensible pages on the calculus of classes and relations.[155]

As Russell wrote in 'Reflections on my Eightieth Birthday' in 1952,

> I wanted certainty in the kind of way in which people want religious faith. I thought that certainty is more likely to be found in mathematics than elsewhere. But I discovered that many mathematical demonstrations, which my teachers wanted me to accept, were full of fallacies, and that, if certainty were indeed to be found in mathematics, it would be a new kind of mathematics, with more solid foundations than those that had hitherto been thought secure.
>
> But as the work proceeded, I was continually reminded of the fable about the elephant and the tortoise. Having constructed an elephant upon which the mathematical world could rest, I found the elephant tottering, and proceeded to construct a tortoise to keep the elephant from falling. But the tortoise was no more secure than the elephant, and after some twenty years of arduous toil, I came to the conclusion that there was nothing more that I could do in the way of making mathematical knowledge indubitable.[156]

Russell had first discovered the joys of mathematics as a teenager, when his elder brother began to teach him Euclid's geometry. He was delighted that mathematics could prove things, but his initial hopes of finding certainty in mathematics were crumbled when he was told that he must accept the axioms as true, assumptions that could not be proved. As he said, it was in mathematics that he had hoped to find indisputable clarity, going on to say, "I hoped that in time there would be a mathematics of human behaviour as precise as the mathematics of machines."[157]

To avoid what he and A. N. Whitehead called a 'vicious circle', he thereby defined a hierarchy of types in which "Whatever involves all of a collection must not be one of the collection." As Morris Kline concisely explains, "Expressed in terms of sets, the theory of types states that individual objects are of type 0; a set of individuals is of type 1; and set of sets of individuals is of type 2; and so forth."[158] Whitehead and Russell therefore said that the proposition "all propositions are either true or false" is meaningless and an illegitimate totality because new propositions cannot be created by statements about 'all propositions'.[159]

The upshot of denying the universal truth of the Principle of Unity was fourfold. First, in denying the validity of the set of all sets, Whitehead and Russell prevented people from mapping the Totality of Existence, further fragmenting the mind and reinforcing people's sense of separation from God, Nature, and each other, none of whom exist as independent beings. Secondly, as paradoxes had appeared in mathematics and logic, ignoring them strengthened the gross distortion in our thinking that Aristotle had established with the seventh pillar of unwisdom, leading to delusion and mental disorder. Thirdly, by denying that logic—the science of thought and reason—is a branch of psychology—the science of mind and consciousness—people were inhibited from studying how concepts are formed and organized in the mind through self-inquiry, necessary if we are to answer the question, "Who are we?" Fourthly, to deny self-referential propositions is to stultify self-reflective Divine Intelligence, which distinguishes human beings from the other animals and machines, like computers, and which enables us to resolve all paradoxes and self-contradictions in Nonduality by looking at both sides of any situation.

## Paradise denied

It is not surprising that Whitehead and Russell never actually completed *Principia Mathematica*, being exhausted by this twenty-year project, and that almost no one read all 2,000 pages of their treatise. One who did was Kurt Gödel, who in 1931 published a landmark paper called 'On Formally Undecidable Propositions of *Principia Mathematica* and Related Systems I'. Gödel was seeking to solve the second of the problems that David Hilbert had posed in 1900: to prove the axioms of arithmetic to be consistent. Hilbert subsequently added another puzzle: to prove that the axioms are complete, that is, that all theorems within the system are provable from the axioms. He suggested that such a proof theory could be developed through what he called metamathematics, a way of talking about mathematics as a formal system of axioms and rules of transformation—expressible in what are essentially meaningless signs—outside the system.

For instance, the expression '2 + 3 = 5' belongs to mathematics, while the statement " '2 + 3 = 5' is an arithmetical formula" is a metamathematical one. By 1930, mathematicians had proved that the tautological propositional calculus and the first-order predicate calculus are both consistent and complete.[160] However, no one had by then proved that the Peano axioms of arithmetic, the Zermelo-Fraenkel axioms of set theory, and the Whitehead-Russell axioms of *Principia Mathematica* are consistent and complete.[161] This is what Gödel set out to prove.

He did so by an ingenious way of mapping metamathematical statements, such as 'Arithmetic is consistent', to arithmetical expressions that evaluate to finite integers. This mapping technique is today called Gödel numbering. Gödel first assigned basic constants, such as '0', '~', and '(', to the odd numbers 1 to 13 and variables of three different types (individuals, such as numbers, classes of individuals, and classes of classes of individuals) the numbers $p^n$, where $p$ is a prime larger than 13 and $n$ is the type of variable, 1, 2, or 3.

He then assigned a formula of $m$ signs to a single number $a$, let us say, calculated as the product of successive primes $p_k$ raised by the Gödel number of each elementary sign, $n_k$:[162]

$$a = \prod_{k=1}^{m} {p_k}^{n_k}$$

For instance, the formula $(\exists x)(x{=}sy)$, meaning every number $y$ has an immediate successor $x$, could be assigned the Gödel number $a = 2^8 \times 3^4 \times 5^{11} \times 7^9 \times 11^8 \times 13^{11} \times 17^5 \times 19^7 \times$

$23^{13} \times 29^{9}$,[163] which is about $1.5*10^{86}$. So Gödel numbers can get pretty big pretty fast. They grow even faster when we look at the concept of proof. Just as a mathematical formula consists of a sequence of signs, a mathematical proof consists of a sequence of formulae, going back to the axioms. So Gödel assigned numbers to proofs, just like formulae.

For instance, the statement $(\exists x)(x{=}s0)$, with Gödel number $b$, is derivable from the first by substituting 0 for $y$, substitution being a basic rule of inference, like Plato's particulars as instances of universals. So the formulae with Gödel numbers $a$ and $b$ are a section of the proof that the number 1 exists (a complete proof would need to go right back to the axioms!). So if this part of the proof were at the beginning, it would be assigned the Gödel number $k = 2^{a} \times 3^{b}$.[164] In general, a proof is assigned a Gödel number calculated as the product of a successive list of primes, each raised to the power of the Gödel number assigned to each statement in the proof. Quite amazing!

Now what is even more amazing is that Gödel then set out to prove the metamathematical statement 'This formula is unprovable'. This statement $G$ with Gödel number $q$ is rather like 'This sentence is false', but with a subtle difference, which does not lead to a contradiction. If 'This formula is unprovable' is not provable, then it is true. Conversely, if $G$ is provable, it is not true. But by Aristotle's Law of Contradiction, if it is true, then it is not provable. Hence $G$ is true if and only if it is *not* provable. As Morris Kline puts it, "the arithmetical statement $G$ is true because it is a statement about integers that can be established by more intuitive reasoning than the formal systems permit."[165]

Gödel then went on to construct the arithmetical statement $A$ that represents the metamathematical statement 'Arithmetic is consistent', proving that $A$ implies $G$. "Hence if $A$ were provable, $G$ would be provable. But since $G$ is undecidable, $A$ is not provable." It is thus not possible to prove the axioms of arithmetic and set theory to be consistent by a method or set of deductive logical principles that can be translated into the system of arithmetic.[166]

In other words, Gödel made a clear distinction between provability and truth; truth is deeper than proof. Provability is an attribute of a mechanistic, linear system of reasoning, while truth is an intuitive, human quality, which machines, like computers, could not understand. In 1961, the philosopher J. R. Lucas wrote a famous article called 'Minds, Machines and Gödel' naturally saying much the same thing, opening with this sentence, "Gödel's theorem seems to me to prove that Mechanism is false, that is, that minds cannot be explained as machines." He based his argument on an intellectual philosophical perspective, rather than a psychological, spiritual, or mystical one based on direct inner knowing of the Divine, using one of his arguments that "human beings are not confined to making deductive inferences."[167]

In other words, Gödel's work shows the invalidity of the fourth pillar of unwisdom, that human beings are machines and nothing but machines. However, some mathematicians and philosophers were horrified by the suggestion that human beings are not just deterministic automata, obeying sets of rigorous rules that could be formally programmed into cybernetic machines, Douglas R. Hofstadter and Daniel C. Dennet calling Lucas's article 'notorious'.[168]

Actually, Gödel's theorems were the first of a number of discoveries that show the limitations of linear reasoning, such as that employed by machines, like computers. In 1936, Alonzo Church and Alan Turing independently extended Gödel's notion that there are undecidable propositions in mathematics, those that can be neither proved nor refuted. In their different ways, they were investigating the capability of mechanistic computability in the horizontal dimension of time. What is now called the Church-Turing thesis states "any calculation that is possible can be performed by an algorithm running on a computer, provided that sufficient time and storage space are available."[169]

Church and Turing were working on the *Entscheidungsproblem*, German for 'decision problem', which went back to the time when Gottfried Leibniz successfully constructed a mechanical calculating machine. Basically, the decision problem asks if there is an algorithm, a mechanical procedure, that can determine whether a particular problem is solvable or not, answering with a yes or no.[170] It does not ask how the problem might be solved if it is solvable; that is another issue.

Church and Turing showed that no such general algorithm exists. In Turing's case, he did this by developing the notion of a universal machine, today called a Turing machine. He then asked the question, "Given a description of a program and its initial input, determine whether the program, when executed on this input, ever halts (completes)."[171] Turing proved that a general algorithm to solve the halting problem for all possible inputs cannot exist.[172]

Church showed, using his lambda calculus, designed to investigate recursive functions, that there is no general algorithm for the decision problem.[173] Turing proved a similar result through his studies of what today is called the Universal Turing Machine.[174] In other words, in linear mathematics, symbolic logic, and computer programming, there are undecidable, incomputable, unprovable, and unsolvable problems, as well as their opposites, which is, of course, an example of the Principle of Unity at work.

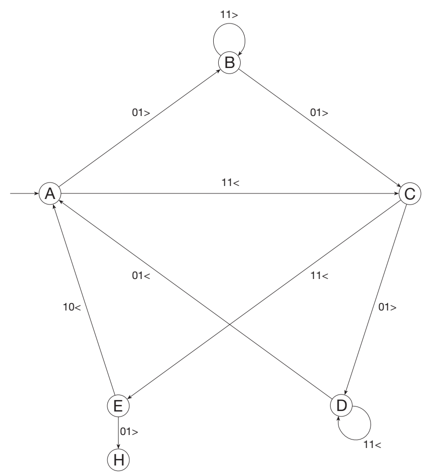


Figure 9.4: *Busy beaver function*

Figure 9.4, shows an example of one of Turing's universal machines, once again showing the ubiquity of mathematical mapmaking, introduced in Section 'Mathematical mapmaking' in Chapter 1, 'Starting Afresh at the Very Beginning' on page 75. Here the nodes are the possible states of the machine, while the arcs are the 'program', the instructions on what the machine should do at each instant in linear time. The Turing machine just consists of a strip of tape that can move left and right and on which symbols are read and written. Each instruction in the program for any particular state is in four parts: read the character at the present position on the tape, write a character, move left or right one position, and change state, all depending on the value of the read character. So the first instruction can simply be expressed as a quintuple: `A 0 1 > B`. This says that when in state A, if 0 is read, write 1, move right, and change to state B.

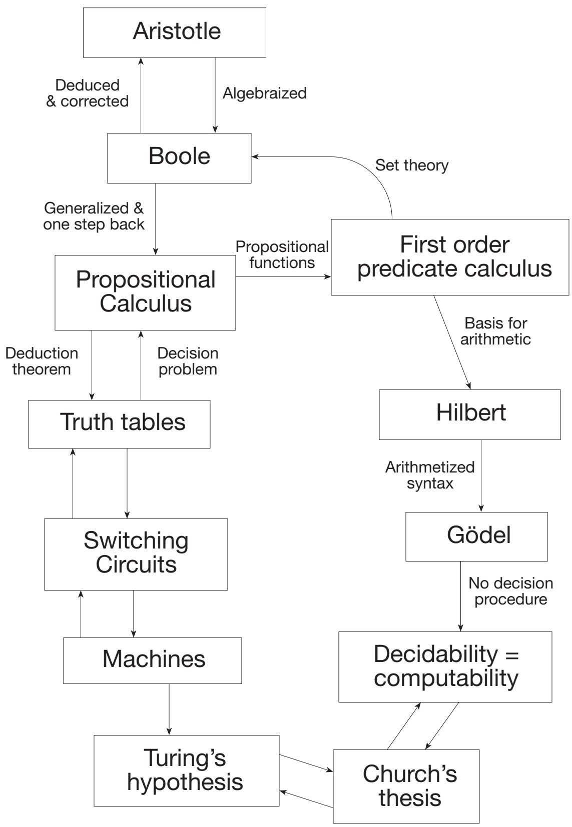



Figure 9.5: *Evolution of either-or formal linear logic*

This particular network is an example of a busy-beaver function, which Tibor Radó devised in 1962 to illustrate the simplicity of a noncomputable function.[175] The purpose of this function in a machine of $n$ states and $k$ symbols is merely to write as many non-blank symbols on a blank tape as possible with as many steps as possible before halting in state H. Because a Turing machine is finite, there is a maximum value for $S(n, k)$ and $\Sigma(n, k)$, the number of steps and symbols for any $n$-$k$ machine, respectively. However, there is no algorithm or decision procedure that can determine these maxima for any particular machine. So since Radó devised this machine, there has been a competition going on among computer scientists to design a record-breaking algorithm for each $n$ and $k$. The example above is the current record holder for a 5-state machine with 2 symbols, giving $\Sigma(5, 2) = 4{,}098$ and $S(5, 2) = 47{,}176{,}870$. Heiner Marxen and Jürgen Buntrock designed this machine in September 1989.[176]

What all these results show is that mechanistic computability, decidability, provability, and solvability are inherently limited. Furthermore, whichever way that the mathematicians have turned, paradoxes have been found in mathematics. To try to resolve this dilemma, mathematicians created four quite different solutions, none of which can be said to provide mathematics with a solid foundation. These are the logical, intuitive, formalist, and set-theoretic schools, each of which is a being in IRL, which means that we do not need to go into them any further.[177]

In summary, Figure 9.5 provides an overview of how either-or formal logic developed in the nineteenth and twentieth centuries, summarizing the West's futile attempts to use linear, mechanistic reasoning to develop a precise language as the basis of our thought processes.[178]

## The Riemann hypothesis

One unsolved problems in mathematics that is, as yet, unsolved, perhaps because it is unsolvable (who knows?), is the Riemann hypothesis, which was the eighth unsolved problem that David Hilbert presented in Paris in 1900, a problem that included Goldbach's conjecture that every even number greater then two is the sum of two primes, also still unsolved.[179] The Riemann hypotheis, proposed by Bernhard Riemann (1826–1866), has been called the 'greatest unsolved problem in mathematics',[180] without realizing that mathematics cannot answer the Big Questions of human existence, such as what is causing mathematicians and scientists to behave as they do?

Nevertheless, to spur mathematicians along, as if they needed such encouragement, in 2000 the Clay Mathematics Institute (CMI) of Cambridge, Massachusetts named seven 'Millennium Prize Problems', awarding one million dollars to the solution of seven unsolved mathematical problems, including the Riemann Hypothesis.[181]

The Riemann hypothesis well illustrates the evolutionary generalizing power of mathematics and that Western thought has now reached an evolutionary cul-de-sac. So let us spend a moment looking at how it has emerged and some attempts to solve it.

We can best begin with Pascal's triangle, although other mathematicians studied this for centuries before him in India, Greece, Iran, China, Germany, and Italy, illustrated in Figure 9.6. These numbers are the binomial coefficients of the polynomial expansion of $(x + y)^n$, more simply expressed as:.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

Figure 9.6: *Pascal's triangle*

$$(x+1)^n = \sum_{k=1}^{n} \binom{n}{k} x^{n-k}$$

where

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

also known as the number of ways of selecting $k$ items from a group of $n$ items in combination theory, wher $n$! is factorial $n$, defined as the product of all the integers up to $n$. So 3! is 6 and 4! is 24.

Now, another fascinating polynomial is the expansion of the power series,[182] studied particularly by Johann Faulhaber (1580–1635), a Rosicrucian collaborator with Johannes Kepler.[183] For instance, as is well known, the sum of the integers from 1 to $n$ is:

$$\sum_{k=1}^{n} k = \frac{1}{2}n(n+1)$$

giving the triangular numbers, 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, …

But what is the polynomial expansion of the general power series?

$$\sum_{k=1}^{n} k^m$$

Well, Faulhaber found expressions for the values of $m$ up to 17,[184] the next three being:

$$\sum_{k=1}^{n} k^2 = \frac{1}{6}(2n^3 + 3n^2 + n)$$

$$\sum_{k=1}^{n} k^3 = \frac{1}{4}(n^4 + 2n^3 + n^2)$$

$$\sum_{k=1}^{n} k^4 = \frac{1}{30}(6n^5 + 15n^4 + 10n^3 - n)$$

Going up to $m$ = 9, this table gives the coefficients for each of the powers of $n$:[185]

| **Power** | ***m*+1** | ***m*** | ***m*-1** | ***m*-2** | ***m*-3** | ***m*-4** | ***m*-5** | ***m*-6** | ***m*-7** |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1/2 | 1/2 | | | | | | | |
| 2 | 1/3 | 1/2 | 1/6 | | | | | | |
| 3 | 1/4 | 1/2 | 1/4 | | | | | | |
| 4 | 1/5 | 1/2 | 1/3 | | -1/30 | | | | |
| 5 | 1/6 | 1/2 | 5/12 | | -1/12 | | | | |
| 6 | 1/7 | 1/2 | 1/2 | | -1/6 | | 1/42 | | |
| 7 | 1/8 | 1/2 | 7/12 | | -7/24 | | 1/12 | | |
| 8 | 1/9 | 1/2 | 2/3 | | -7/15 | | 2/9 | | -1/30 |
| 9 | 1/10 | 1/2 | 3/4 | | -7/10 | | 1/2 | | -3/20 |

Table 9.12: *Polynomial coefficients of expansion of sum of powers*

There seems to be a pattern here, but what on earth is it? The coefficients total one, the first being 1/($m$+1) and the second ½. The third coefficient has a value, whose pattern is far from clear. After this the alternating coefficients are zero and the other coefficients alternate from minus to plus. But does this pattern continue indefinitely and what is the pattern that underlies the coefficients? Such a puzzle is not unlike the intelligence tests that teachers set children at school or those that Mensa sets as entry to their exclusive club. Well, like Tycho Brahe, measuring the positions of the stars and planets, Faulhaber did not find the underlying pattern. It was left to Jacob Bernoulli (1654–1705), acting like Kepler to Tycho, whose story we tell on page 916 in Subsection 'The first scientific revolution', to find a generalized expression for these coefficients. Here it is:[186]

$$\sum_{k=1}^{n} k^m = \frac{1}{m+1}\sum_{j=0}^{m}(-1)^j\binom{m+1}{j}B_j n^{m+1-j}$$

where $B_j$ is a Bernoulli number, defined recursively:

$$B_j = -\sum_{i=0}^{j-1}\binom{j}{i}\frac{B_i}{j-i+1}$$

with $B_0 = 1$. Isn't that amazing? Here are the first few Bernoulli numbers:[187]

| **Number** | $B_0$ | $B_1$ | $B_2$ | $B_4$ | $B_6$ | $B_8$ | $B_{10}$ | $B_{12}$ | $B_{14}$ | $B_{16}$ | $B_{18}$ | $B_{20}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Value** | 1 | -1/2 | 1/6 | -1/30 | 1/42 | -1/30 | 5/66 | -691/2730 | 7/6 | -3617/510 | 43867/798 | -174611/330 |

These apparently haphazard numbers, which get larger and larger in absolute terms, are of such central importance in mathematics, Ada Lovelace showed how they could be calculated with Charles Babbage's Analytical Engine, turning Babbage's formulae into tabular form, published at the end of her memoir to Menabrea's 'Sketch of the Analytical Engine' in 1843. Not surprisingly, she did not do so without considerable effort, saying in a letter to Babbage, "I am in much dismay at having got into so amazing a quagmire & botheration with these *Numbers*."[188] This was the first program ever published, much more complex than the initial programs that ran on the first stored-program computers over a century later. Ada has thus been called the world's first programmer, although she was clearly much assisted by Babbage himself.

Now the next step on this process of generalization in mathematics was to consider the power series where $m$ is negative, which gives the possibility that even the sum of an infinite series of such terms converges to a finite value. The general formula here is called the Riemann zeta function, which Euler showed could also be expressed as the product of terms involving just prime numbers:

$$\zeta(s) = \sum_{n=1}^{\infty}\frac{1}{n^s} = \prod_{k=1}^{\infty}\frac{1}{1+\frac{1}{p_k^s}}$$

where $p_k$ is the $k$th prime.

John Derbyshire calls this amazing relationship the 'Golden Key',[189] which causes mathematicians to go all a flutter. For primes are the atoms of number theory, all integers being uniquely expressible as the product of prime numbers—the fundamental theorem of arithmetic. But no pattern has been found in the distribution of the primes other than the prime number theorem (PNT), which states that if a random integer is selected in the range of zero to some large integer $N$, the probability that the selected integer is prime is about $1 / \ln(N)$, where $\ln(N)$ is the natural logarithm of $N$.[190]

One significant consequence of Euler's product, as it is called, is that for $n \geq 0$[191]

$$\zeta(2n) = (-1)^{n+1}\frac{B_{2n}(2\pi)^{2n}}{2(2n)!}$$

For instance, for $n = 1$, we have, a result that Euler, himself,[192] found:

$$\frac{2^2}{2^2+1} \times \frac{3^2}{3^2+1} \times \frac{5^2}{5^2+1} \times \frac{7^2}{7^2+1} \times \frac{11^2}{11^2+1} \times \ldots = \frac{\pi^2}{6}$$

So we have a surprising relationship between the prime numbers and $\pi$, the ratio of the circumference of a circle to its diameter, just one other example where $\pi$ pops up in the most unexpected places.

Some other consequences of the zeta function fascinate mathematicians when $s$ is negative. Considering just integer values, we have this formula:

$$\zeta(-n) = -\frac{B_{n+1}}{n+1}$$

As $B_n$ is zero for all odd values of $n$, the zeta function is zero for all even negative integers, known as the trivial zeros. Considering $s$ as a real number, using a more general formula for the zeta function, between these zero points, the function is continuous, swinging increasingly as $s$ grows negatively, being positive between $s = -2(2n-1)$ and $-4n$ and negative otherwise, where $n > 0$, illustrated in Figure 9.7.

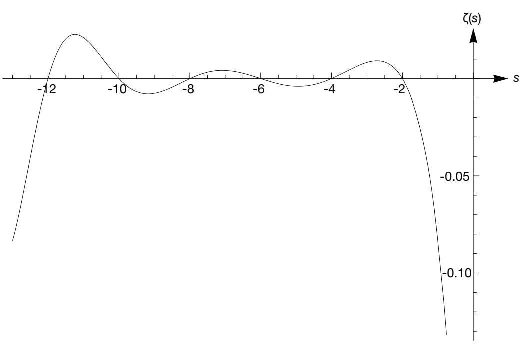


Figure 9.7: *Trivial zeros in Riemann zeta function*

But things get really interesting when $s$ is a complex number of the form $\sigma + it$ It is not easy to visualize the way that the zeta function behaves with complex $s$ as this requires many years of practice,[193] for it requires four dimensions to plot the real and imaginary inputs and outputs from the function.[194] Furthermore, the non-trivial zero points do not have the regular pattern of the trivial zero points, other than that the first few that Bernhard Riemann found lay on a line ½ + $it$, the first three being (½, 14.134 725 …), (½, 21.022 040 …), and (½, 25.010 856 …).[195] In a paper published in 1859, he therefore hypothesized that all non-trivial zero points lie on this line. This is the Riemann Hypothesis.

In 1900, when Hilbert included the Riemann Hypothesis in his list of unsolved problems in mathematics, it was known that all zero points would fall in a critical strip where $0 < \sigma < 1$, with a critical line at $\sigma = ½$.[196] Then in 1914, G. H. Hardy proved that there are an infinite number of zeros on this critical line.[197] But he did not prove that there are none outside the line and no one has done so since.

However, in 1973, Hugh L. Montgomery, noting that the zero points line up in relatively uniform intervals, far more regular than the primes themselves, found that the differences between the zeros seem to have a distribution given by this formula:[198]

$$1-\left(\frac{\sin \pi u}{\pi u}\right)^2$$

Now the year before, Montgomery had met Freeman Dyson by chance at the Princeton Institute of Advanced Studies, the latter pointing out that Montgomery's pair correlation conjecture has the same form as the distribution function of the energy levels of subatomic particles.[199] This really got the mathematicians excited, for this similarity seems to indicate a link between the distribution of the prime numbers and quantum physics.

Then in 1996, Alain Connes pointed out another surprising relationship: between his non-commutative geometry, for which he was awarded the Field's Prize, and the Riemann function. This connection opened up a quite new approach to proving the Riemann hypothesis, leading some to speculate that non-commutative geometry could form the basis for the discovery of the fundamental law of nature, one that could explain the creation of the universe. As the commentator on *The Cosmic Code Breakers,* a 2011 television programme on the Riemann hypothesis enthusiastically proclaimed, as the new geometry is closely related to prime numbers, if the secrets of the primes are clarified using non-commutative geometry, then the theory of everything would be solved. The century-long search for the hidden meaning behind the prime numbers could well turn out to be the theory of everything, the Creator's blueprint for the Universe.[200]

Not all mathematicians share this enthusiasm. An anonymous mathematician who doesn't has said, "What Connes has done, basically, is to take an intractable problem and replace it with a different problem that is equally intractable."[201] So it might be that while the Riemann hypothesis could be a true theorem of mathematics, it is not one that can be proved using any mathematical tool, much as Gödel indicated.

Now while this is obviously great fun, for many enjoy searching for simple patterns underlying the world we live in, such pastimes cannot lead us back to Reality. And neither can conventional scientific method, whose evolution we look at next.

## The evolution of scientific method

As the way we think and reason determines our behaviour, you might think that science would be interested in addressing this issue. Apparently not so. Despite the great successes that science has made during the past few centuries, there is one question that neither mainstream reductionist science nor its holistic alternative can answer satisfactorily: "what is causing the pace of evolutionary change to accelerate exponentially?" The reason for this is not only because of the assumptions that science makes about the nature of reality; it is also because of the limitations of scientific method itself.

So to understand where we human beings have come from and where we are all heading in such a frantic rush, we need to allow scientific method, itself, to evolve. In this way we can see that while the Unified Relationships Theory is revolutionary in the context of Western civilization, it is nevertheless still scientific. To do this, we need to agree a definition of science.

For me, science is simply a coherent body of knowledge that corresponds to our all experiences whatever they might be and whoever might have them. This means that if our experiences are limited, so is our science. Furthermore, if our knowledge is fragmented, while parts might be cohesive and therefore scientific, the whole cannot be.

Today, what is commonly called science is both fragmented and limited. So until we remove the constraints that we place on our learning, we cannot say that our knowledge is truly scientific. Most particularly, we shall continue to manage our business affairs having very little understanding of what we are doing, a situation that can only lead to catastrophe within a few years.

So how have we reached the perilous situation that we are in today? Well, let us take it that formal science began with Aristotle. As I understand the situation, Aristotle had no conscious method in his scientific inquiries. Starting with some assumptions or axioms, he simply made observations of the world around him through his physical senses and drew conclusions. Apparently, Aristotle did not see the need to test his deductions by experimentation.

This situation began to change in the thirteenth century with Roger Bacon, an English philosopher and Franciscan. It seems that Bacon was the first European to see the need to base our learning on direct experience, rather than the rational deductive methods that the world of learning had inherited from Aristotle.

As such, Bacon was widely known and respected throughout Europe as the *Doctor Mirabilis* (Wonderful Teacher), both for his methods and his discoveries, and for his boundless energy in developing and expressing his ideas.[202]

This situation began to change in 1257, when he was about 37. In that year, Bacon joined a religious order of friars. But his reforming zeal and contemptuous disposition did not go down well with his superiors, who did their best to constrain him.

Bacon felt aggrieved by this behaviour because he thought that his experimental methods served to confirm the Christian faith. So he appealed to the Pope for support. It seems that what Bacon was proposing was a vast encyclopædia of all the known sciences, a project that would be coordinated by a papal institute. So not only was Bacon emphasizing the empirical nature of human knowledge, he could also see the need for the coherence of all our knowledge, principles that are central to the URT.

However, the Pope apparently misunderstood Bacon's proposals, thinking that the project was already far advanced. So the Pope requested to see the results of the project that he assumed that Bacon had been conducting. This put Bacon in a bit of a predicament. Having no other choice, he set out to complete this project on his own, working in secret by papal command without the knowledge of his superiors, a situation that is not unlike my own endeavours to integrate all knowledge into a coherent whole.

Inevitably, these exertions affected Bacon's ability to participate fully in the activities of the friary, which did not please his superiors too much. Eventually, around 1278 he was condemned to prison for "suspected novelties" in his teaching, an example of the challenges faced by evolutionary pioneers within a fearful environment that seeks to restrict creativity.

The next major development in scientific method that we need to consider was introduced by Bacon's namesake, Francis Bacon some 350 years later. I just want to mention two points. Bacon was concerned with two major issues, pure and applied science, the development of knowledge for its own sake and the application of this knowledge for "the relief of man's estate".[203]

In other words, Bacon was the first to put into words the belief that it is the purpose of science to exploit Nature for the selfish desires of human beings. Of course, such a belief could only arise in the West, which is both intellectually and often experientially separate from our Divine Source. Today, this belief has led to ecological devastation, which is leading to the extinction of the human race before we have had the opportunity to realize our fullest potential as a species.

The other major contribution that Bacon made to scientific method was the principle of induction. This concept was necessary in order to describe the essence of the experimental method, just then being fully utilized by Bacon's contemporary, Galileo Galilei. Bacon described the inductive method in Book II of *Novum Organum*, published in 1620. The title of this book is a reference to Aristotle's *Organon*, in which Aristotle had introduced the deductive method of reasoning around two thousand years earlier.

In the *Advancement of Learning*, published in 1605,[204] Bacon argued vigorously "Aristotle's logic was entirely unsuitable for the pursuit of knowledge in the 'modern' age. Accordingly, *The New Organon* propounds a system of reasoning to supersede Aristotle's, suitable for the pursuit of knowledge in the age of science."[205]

The principle of induction in science, not to be confused with induction in mathematics,[206] is very simple. It can be defined as follows:

> If a large number of As have been observed under a wide variety of conditions, and if all those observed As without exception possessed the property B, then all As have the property B.[207]

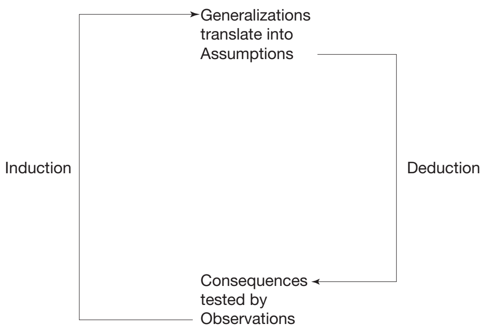



Figure 9.8: *Induction and deduction*

The principle of induction thus leads to generalized statements, from which predictions about particular situations can be deduced. Figure 9.8 shows the cyclical relationship of induction to deduction, indicating that induction does not actually start from observation.[208] This is what A. F. Chalmers calls 'naive inductionism'. For in practice all observation statements are theory dependent. It is not possible to observe anything without some preconceptions of what is being observed.

It was the eighteenth-century Scottish philosopher, David Hume, who first pointed out this serious weakness of the inductive method. If science is to produce certain knowledge, these generalizations need to be true for all time. He raised two problems with this assumption of science, the first logical and the second psychological, which are discussed by Karl Popper. The first of these problems is:

> Are we justified in reasoning from [repeated] instances of which we have experience to other instances [conclusions] of which we have no experience?[209]

The answer is no, however great the number of repetitions. For instance, for those of us who live between the Arctic and Antarctic circles, the sun rises every day, even though on some occasions we don't see it because it is hidden by clouds. But is it reasonable to assume that this process will continue indefinitely? Obviously not. The physicists have estimated that in some four to five billion years the Sun will die along with the Earth.[210] So one day, there will be neither a sunrise nor anyone around to observe it.

David Hume goes on to pose his psychological problem of induction:

> Why, nevertheless, do all reasonable people expect, and believe, that instances of which they have no experience will conform to those of which they have experience? That is, why do we have expectations in which we have great confidence?[211]

His answer to this problem, interpreted by Karl Popper, is:

> Because of 'custom or habit'; that is, because we are conditioned, by repetitions and by the mechanism of the association of ideas; a mechanism with which, Hume says, we could hardly survive.[212]

Hume's attack on empiricism evidently caused a major crisis in the scientific community, for he was questioning the very basis of scientific reasoning. Russell highlighted the issue when he said:

> It is therefore important to discover whether there is any answer to Hume within the framework of a philosophy that is wholly or mainly empirical. If not, there is no intellectual difference between sanity and insanity. The lunatic who believes that he is a poached egg is to be condemned solely on the grounds that he is a minority, or rather—since we must not assume democracy—on the grounds that the government does not agree with him. This is a desperate point of view, and it must be hoped that there is some way of escaping it.[213]

Popper provided the most generally accepted way of escaping the scientific problem of induction. He proposed that while scientific generalizations could not be verified by repeated repetition, they could be falsified. This approach to scientific discovery has had many adherents.

However, A. F. Chalmers has pointed out that this approach is flawed. He states, "Theories cannot be conclusively falsified because the observation statements that form the basis of falsification may themselves prove to be false in the light of later developments."[214] For all observation statements are theory dependent, and when theories change, these observation statements may possibly change.

This is what Chalmers calls 'naive falsificationism'. A more sophisticated approach, proposed by Popper himself, is to view scientific discovery in an evolutionary manner. In this view, Popper called scientific theories or hypotheses 'conjectures'.[215] Science advances by making conjectures that can either be confirmed or falsified by observation. Most particularly, if a bold conjecture can be confirmed or a cautious one falsified, then science can progress. In contrast, as Chalmers points out, "little is learnt from the falsification of a bold conjecture or the confirmation of a cautious conjecture".[216]

However, even this account of scientific method does not satisfactorily describe what happens when science makes one of its major breakthroughs, the classic example being the scientific revolution begun by Copernicus in 1543 with his *Book of the Revolutions of the Heavenly Spheres* and completed by Isaac Newton in 1687 with his *Mathematical Principles of Natural Philosophy*.

When studying this development, Thomas S. Kuhn pointed out that scientific theories need to be seen as a complex structure of concepts, which he famously called 'paradigms' from the Greek word paradeiknumi meaning 'show side by side'. From this, he made a clear distinction between normal science, which works within the context of a particular paradigm, and scientific revolutions, when a radical change is made to the conceptual structures that guide scientific research.

This is what generally happens in what Thomas S. Kuhn called normal science:

> … 'normal science' means research firmly based upon one or more past scientific achievements, achievements that some particular scientific community acknowledges for a time for its further practice.[217]

However, such an approach to science does not satisfactorily describe the process that Copernicus, Kepler, Galileo, and Newton went through in the sixteenth and seventeenth centuries or that of Priestley and Lavoisier in developing the oxygen theory of combustion.[218]

By looking at such examples in the history of scientific discovery, Kuhn saw that such a radical change in world-view comes about as the result of anomalies in the overall structure of existing scientific theories; experience no longer matches the theory, leading to what Kuhn called a paradigm shift or change. Such a transformation is the essence of scientific revolutions, which he described thus:

> … at times of revolution, when the normal scientific tradition changes, the scientist's perception of his environment must be re-educated—in some familiar situations he must learn to see a new gestalt.[219]

Kuhn went on to say that it is as much the consensus of scientific communities that decides what paradigms should be used as rational argument. In other words, Kuhn asserted that science is as much a social activity as an objective, rational process. This observation of the world as it is was not too popular in some quarters. For instance, Imre Lakatos did not like what philosophers call 'relativism', although Kuhn denied that he was a relativist.[220] While supporting the notion that scientific theories are structures, Lakatos sought a way of restoring both rationalism and absolutism to science.

He attempted to do this with the concept of a 'hard core' that scientific research programmes should adhere to. "The hard core of a programme … takes the form of some very general theoretical hypotheses from which the programme is to develop."[221] For instance, "The hard core of Newtonian physics is comprised of Newton's laws of motion plus his law of gravitational attraction."[222] Most particularly, "any scientist who modifies the hard core has opted out of that particular research programme," typically being ostracized by her or his colleagues. It is therefore not surprising that scientists with a spiritual or even mystical orientation have been very careful to keep their experiences secret.

The next player in this game to appear was Paul Feyerabend. Feyerabend was concerned that these hard core paradigms and methods could inhibit the growth of scientific knowledge. In *Against Method*, he therefore proposed an anarchistic approach to learning in which "anything goes".[223]

Most particularly, he wanted to challenge the claim that scientific method is superior to any other method of developing knowledge about ourselves and the world we live in. For if science is to play its full part in the world, we need to look at it in the context of the social environment in which it is taking place. As Feyerabend said, we need to "free society from the strangling hold of an ideologically petrified science just as our ancestors freed us from the strangling hold of the One True Religion!"[224]

In other words, as a growing number of scientists are beginning to realize, if humanity is to resolve the great crisis it is facing at the present time, we need to free science of scientism, a generally derogatory term indicating a belief in the omnipotence of scientific knowledge and techniques.

We can begin to do this by noting that one of the most fundamental assumptions of science is false, articulated by A. F. Chalmers, "I accept, and presuppose throughout this book, that a single, unique, physical universe exists independently of observers".[225] Nor is this all. Karl Popper believed that there is such a thing as objective knowledge without a knowing subject, a belief that shows how far Western philosophy and science has departed from Reality.

This brief history of the struggle to find a sound basis for scientific method overlooks another approach to scientific reasoning, that of hypothesis or abduction, terms introduced by Charles S. Peirce in the 1800s, as we see in Table 1.2, "Approaches to scientific method," on page 123. But, again, abductive reasoning does not lead us to the Absolute Truth, to an understanding of what is causing the pace of technological development to accelerate exponentially.

In order to overcome the problem of scientism and in his attempts to integrate science and religion, Ken Wilber has introduced a radically new approach to scientific method. Following St Bonaventure and Hugh of St Victor,[226] Ken points out that we human beings have at least three modes or eyes of attaining knowledge: "the eye of flesh, by which we perceive the external world of space, time, and objects; the eye of reason, by which we attain knowledge of philosophy, logic, and the mind itself; and the eye of contemplation, by which we rise to a knowledge of transcendent realities".[227]

Ken then goes on to assert that the same scientific method can apply to each of these three eyes, what he calls "the three strands of all valid knowing":

1. Instrumental injunction. This is an actual practice, an exemplar, a paradigm, an experiment, an ordinance. It is always of the form, 'If you want to know this, do this'.
2. Direct apprehension. This is an immediate experience of the domain brought forth by the injunction; that is, a direct experience of apprehension of data (even if the data is mediated, at the moment of experience it is immediately apprehended). William James pointed out that one of the meanings of 'data' is direct and immediate experience, and science anchors all of its concrete assertions in such data.
3. Communal confirmation (or rejection). This is a checking of results—the data, the evidence—with others who have adequately completed the injunctive and apprehensive strands.[228]

Each of these ideas has made a significant contribution to the establishment of a rational way of thinking and learning that can produce a true representation of ourselves and the world we live in. Yes, we need experimentation, yes, scientific theories are structures, yes, there is a danger here that these structures might inhibit our learning, and yes, we need to apply our scientific inquiries to our physical, mental, and spiritual domains, all three.

However, as they stand at the moment, all these different approaches lack the cohesion of Integral Relational Logic. Taking Ken's three eyes of knowing, in particular, he is using his analytical powers to distinguish these different ways of developing knowledge without recognizing that these concepts are subclasses of **Being**, the superclass of all our learning.

Furthermore, why does Ken only accept knowledge as valid that has been confirmed by a consensus? As Alexis de Tocqueville[229] and John Stuart Mill[230] showed in the middle of the nineteenth century, democracies can be tyrannous. So what happens when an individual is a pioneer, exploring ways of learning that have never been tried before? Does this invalidate the experiment if no others in society are yet ready to repeat this experiment in learning?

## The surface of things

As axiomatic mathematical proof, deductive logic, and generally accepted scientific methods cannot lead us to Wholeness and the Truth, cannot provide us with a valid picture of the world we live in, it is not surprising that science and medicine, concerned only with the superficial, have also reached an evolutionary cul-de-sac.

To give but one example, scientists assert that they "have found that everything in the Universe is made up from a small number of basic building blocks called elementary particles, governed by a few fundamental forces," as CERN's website tells us.[231] This atomistic philosophy has a long history, going back, once again, to the ancient Greeks, to Leucippus and Democritus some 2,400 years ago. As *Encyclopædia Britannica* tells us, it was Democritus who named the "infinitely small building blocks of matter atomos, meaning literally 'indivisible', about 430 BC", articulating the beliefs of his teacher, Leucippus. (The Greek verb 'to cut' was temnein, the substantive being tomos.)

Even though Ernest Rutherford showed in 1911 that the atom is not actually indivisible, but consists of a nucleus and orbiting electrons, the belief persists in the existence of a fundamental particle that cannot be further subdivided. Indeed, this belief is so strong among the 13,000 particle physicists around the world that they have persuaded governments to build them multimillion-dollar particle accelerators, which they use to study the properties of and interactions between the multitude of subatomic particles that have been discovered in the past one hundred years. At the time of writing, the hunt is on for a 'Higgs boson', supposedly a particle or set of particles that give everything in the physical universe, including us, mass.

For instance, Stephen W. Hawking was reported as saying on BBC radio in December 2006, “scientists still have ‘some way to go’ to reach his prediction in his bestselling *A Brief History of Time* that mankind would one day ‘know the mind of God’ by understanding the complete set of laws which govern the universe.”[232] He still believes that the giant LHC atom smasher that went into operation in the CERN nuclear physics laboratory in Geneva in 2008 and then broke down is necessary to reveal these laws, which he thinks could be developed within twenty years. Furthermore, he still believes that “Mankind will need to venture far beyond planet Earth to ensure the long-term survival of our species,” not recognizing that the human race is not immortal; it is subject to the same laws as any other structure in the Universe.

There seems to be no limit to this tomfoolery. For as soon as one group of scientists claim to have found the ultimate particle, another group will come along to try to prove them wrong.[233] There is no end to this process. It is quite clear that studying physics cannot lead us to Wholeness and the Truth. Because scientists do not accept a holistic science of reason that truly describes how human beings think and learn, they are still leading both politicians and the general public astray.

**FERMIONS** matter constituents spin = 1/2, 3/2, 5/2, ...

| Leptons spin = 1/2 | | | Quarks spin = 1/2 | | |
|---|---|---|---|---|---|
| Flavor | Mass GeV/$c^2$ | Electric charge | Flavor | Approx. Mass GeV/$c^2$ | Electric charge |
| $\nu_e$ electron neutrino | $<1\times10^{-8}$ | 0 | **u** up | 0.003 | 2/3 |
| **e** electron | 0.000511 | –1 | **d** down | 0.006 | –1/3 |
| $\nu_\mu$ muon neutrino | <0.0002 | 0 | **c** charm | 1.3 | 2/3 |
| $\mu$ muon | 0.106 | –1 | **s** strange | 0.1 | –1/3 |
| $\nu_\tau$ tau neutrino | <0.02 | 0 | **t** top | 175 | 2/3 |
| $\tau$ tau | 1.7771 | –1 | **b** bottom | 4.3 | –1/3 |

Figure 9.9: *Part of standard model of fundamental particles and interactions*

Yet it is interesting to note that the standard model of fundamental particles and interactions published by the Contemporary Physics Education Project (CPEP) contains tables just like the basic construct in Integral Relational Logic. Figure 9.9 shows just one of these tables, indicating that all of us, including the particle physicists, use IRL in organizing our ideas. Even in physics, mathematical measurement is secondary to semantic structures.

At the other end of the scale, scientists are searching for the origin of the Universe and forms of life in outer space. It is a fundamental misconception to think that we shall “unlock the secrets of the universe” and discover the origins of humanity by sending multibillion-dollar telescopes into the sky, which is a primary goal of NASA’s Origins Program using the Hubble Space Telescope.[234] We can only discover who we truly are as human beings through self-inquiry, by turning the attention inwards rather than outwards. And this endeavour does not cost a cent or a penny.

We can also see that there is no point in searching for life on Mars or anywhere else in outer space. For instance, the mission of the SETI (Search for Extraterrestrial Intelligence)

Institute is "to explore, understand and explain the origin, nature and prevalence of life in the universe".[235] But life is not 'out there'. The search for extraterrestrial intelligence is thus doomed to fail because any hypothetical intelligent being in another part of the physical universe would know that Intelligence is divine, and would not bother trying to communicate with beings who did not know this.

## Diving beneath the surface

If we are to escape from the evolutionary cul-de-sac that modern science, mathematics, and logic have led us into, we need to dive beneath the material surface of our lives and look into the depths of the Cosmic Psyche. We need to escape from the prison cells that our egoic minds have incarcerated us in. David Bohm, a friend and colleague of Albert Einstein in the 1940s and 50s, began to show us how scientists can pursue this path as well as the mystics.

Like Einstein, he was particularly interested in Wholeness, not only to solve the mysteries thrown up by the incompatibilities of modern physics, but also because Wholeness is essential in solving our immense social problems. As Bohm said,

> The widespread and pervasive distinctions between people (race, nation, family, profession, etc., etc.), which are now preventing mankind from working together for the common good, and indeed for survival, have one of the key features of their origin in a kind of thought that treats things as inherently divided, disconnected, and 'broken up' into yet smaller constituent parts. Each part is considered to be essentially independent and self-existent.[236]

Regarding the two primary theories in physics, he said,

> Relativity and quantum theory agree, in that they both imply the need to look on the world as an undivided whole, in which all parts of the universe, including the observer and his instruments, merge and unite in one totality. In this totality, the atomistic form of insight is a simplification and abstraction, valid only in some limited context.[237]

In contrast, Bohm had this to say about his scientific colleagues:

> Most physicists still speak and think, with an utter conviction of truth, in terms of the traditional atomistic notion that the universe is constituted of elementary particles which are 'basic building blocks' out of which everything is made. In other sciences, such as biology, the strength of this conviction is even greater, because among workers in these fields there is little awareness of the revolutionary character and development in modern physics. For example, modern molecular biologists generally believe that the whole of life and mind can ultimately be understood in more or less mechanical terms, through some kind of extension of the work that has been done on the structure and function of DNA molecules. A similar trend has already begun to dominate psychology. Thus we arrive at the very odd result that in the study of life and mind, which are just the fields in which formative cause acting in undivided and unbroken flowing movement is most evident to experience and observation, there is now the strongest belief in the fragmentary approach to reality.[238]

In endeavouring to make sense of the paradoxes of quantum physics, Bohm noticed that in "looking at the night sky, we are able to discern structures covering immense stretches of space and time, which are in some sense contained in the movements of light in the tiny space encompassed by the eye."[239] He saw this as evidence of "a total order … contained, in some implicit sense, in each region of space and time."[240] This led him to realize the existence of an enfolded or implicate order, in contrast to the explicate order, which the laws of physics that thus far mainly referred to.[241] In contrast, Bohm proposed that to formulate the laws of physics "primary relevance is to be given to the implicate order, while the explicate order is to have a secondary kind of significance."[242]

Bohm used some physical analogies to explain what he meant:

> A more striking example of implicate order can be demonstrated in the laboratory, with a transparent container full of a very viscous fluid, such as treacle, and equipped with a mechanical rotator that can 'stir' the fluid very slowly but very thoroughly. If an insoluble droplet of ink is placed in the fluid and the stirring device is set in motion, the ink drop is gradually transformed into a thread that extends over the whole fluid. The latter now appears to be distributed more or less at 'random' so that it is seen as some shade of grey. But if the mechanical device is now turned in the opposite direction, the transformation is reversed, and the droplet suddenly appears, reconstituted.[243]

Bohm also uses the hologram as an illustration of undivided wholeness, from the Greek holo 'whole' and *gramma* 'writing', related to grapho 'to write'. "Thus the hologram is an instrument that, as it were, 'writes the whole'."[244] When the image of an object is created on a photographic plate using a laser beam, there is no one-to-one correspondence between parts of the illuminated object and parts of the image of this object on the plate. Rather, the interference pattern on each region R of the plate is relevant to the whole structure.[245] Furthermore, Bohm likened his view of a holographic universe to Karl Pribram's view of the holographic brain.

> Pribram has given evidence backing up his suggestion that memories are generally recorded all over the brain, in such a way that information concerning a given object or quality is not stored in a particular cell or localized part of the brain but rather that all information is enfolded in the whole. This storage resembles a hologram in function.[246]

The theory of the implicate order is also central to the reconciliation of the incompatibilities between relativity and quantum theories: "Relativity theory requires continuity, strict causality (or determinism) and locality. On the other hand, quantum theory requires noncontinuity, noncausality, and nonlocality."[247] Bohm illustrated the relationship between relativity and quantum theories with two cameras at right angles pointing at a fish swimming in a tank, reproduced in Figure 9.10.[248] The television screens linked to cameras A and B show different images of one underlying reality. It is the profound implicate order that is primary; the superficial explicate order of our senses that we look at through our television sets is secondary.

To give this underlying, undivided reality some substance, Bohm introduced the notion of the holomovement, which he liked to an undivided flowing stream, whose substance is

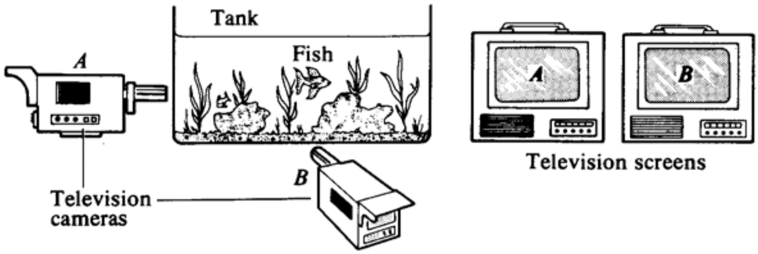



Figure 9.10: *Opposite perspectives of one underlying reality*

never the same, along the lines of Heraclitus, who said, "You cannot step twice in the same river."[249] He also saw this view as a development of A. N. Whitehead's process view of reality.[250] As he said, "On this stream, one may see an ever-changing pattern of vortices, ripples, waves, splashes, etc., which evidently have no independent existence as such. Rather, they are abstracted from the flowing movement, arising and vanishing in the total process of flow."[251] Bohm then went on to say, "Everything is to be explained in terms of forms derived from this holomovement. Though the full set of laws governing its totality is unknown (and, indeed, probably unknowable)."[252]

This statement is very close to the Truth, but not quite. The holomovement still encapsulates the concept of linear time, which we need to transcend if we are to be truly liberated from the bondage of past and future. We can do this by allowing the river to flow into the ocean of Consciousness, a vast ball of water whose origin is the centre of the ocean, illustrated in Figure 4.5 on page 256. In a similar fashion, the quantum physicist Amit Goswami regards Consciousness as the primary reality, but there is no mention of the holomovement or the implicate order in his book, *The Self-Aware Universe: How Consciousness Creates the Material World*.

This view of Consciousness as Reality is somewhat different from that of some other physicists. For instance, Danah Zohar describes underlying reality as a quantum vacuum, the 'well of being'.[253] Nevertheless, she goes on to say, "The quantum vacuum is very inappropriately named because it is not empty. Rather, it is the basic, fundamental and underlying reality of which everything in this universe—including ourselves—is an expression."[254]

Another physicist, Mark Comings, has similarly said, "This Quantum Vacuum is more aptly named the Quantum Plenum,"[255] the Latin neuter of plenus 'full'. He associates the quantum plenum with space, which he says has virtually unlimited potential locked up within it.[256] It seems that by saying that Ultimate Reality is empty, the physicists have been attempting to associate their scientific world-view with the central concept of Buddhism: *shunyata*, 'emptiness or void'. Yet, Reality, as the union of all opposites, is both Emptiness and Fullness. However, it is vitally important not to be confused by the parallels between quantum physics and Eastern mysticism. Reality is neither space nor time, even though Consciousness has some of the properties of space discovered by physicists.

We can see this most clearly with David Bohm's theory of the implicate order, even though Bohm himself did not completely transcend his conditioning as a physicist.[257] The Unified Relationships Theory embraces the implicate and explicate orders by noticing that structures have both a surface, accessible to our senses, and a depth, which we can call the structure's essence, from the Latin word esse meaning 'to be', which determines their essential nature. The essence of structures can easily be demonstrated with the collection of A's in thirty different fonts in Figure 9.11.[258] We human beings can see that there is a certain 'A-ness' about these characters, which enables us to see the commonality amongst them, different as they are.

Figure 9.11: *Illustration of the essence of structures*

However, when I ran an experiment to see how many of these A's my optical character recognition (OCR) program would recognize, it managed only twelve: 40%. I suspect that even the most advanced OCR program would have difficulty in reading all these A's. The reason for this is that these forms have a deep underlying essence, which resonates with our understanding of what the letter A looks like. We can immediately see forms as wholes, without any need for pattern recognition algorithms, which computers must resort to.

As it is with simple letters, so it is with human faces, which we are able to recognize without any difficulty, complex as they are. In music, poetry, art, literature, etc., it is the essence of these structures that evoke beautiful feelings within us. They cannot be fully appreciated with the intellect, even though the mind likes to analyse these structures to see how a piece of music, for instance, is composed. Analysing structures destroys their essence, which provides us with meaning and value. The essence of structures is not something that can be quantified in monetary terms, for instance. As the saying goes, "The best things in life are free."

This is nowhere clearer than when we are in the wilderness, communing with Nature. For instance, the trees in the forests of Scandinavia are not just there to make houses, furniture, and paper. We can feel the presence of God deep in the forest, far away the madness of the world we live in today.

Going even deeper, all these feelings show quite clearly that all sentient beings have a living essence, called 'the soul' in human beings, which determines our uniqueness. This does not mean that the soul survives death or is reincarnated. For the soul, like everything else in the world of form, is just an abstraction from Consciousness, with no separate existence. Beyond the soul are the female and male principles, which we share with others of the same sex. Ultimately, the Essence of the Universe as a whole is the Absolute, which we can most simply call Love, for God is Love, as John wrote in his first epistle.[259]

This is nowhere clearer than when a woman and man love each other unconditionally. For in their divine lovemaking, two become one beyond all thoughts, the most beautiful meditation that any of us can engage in. These experiences show that we human beings can love each other not only as woman and man, but also as goddesses and gods. For in Reality, there is no separation between the Divine and human. God is everywhere and everywhen, in every nook and cranny. And when we know this deep in our hearts, there is no need for CNN to broadcast such programmes as *God's Warriors*, broadcast in August 2007. All holy wars—wars about the Whole—with then have come to an end and we can live in Peace, perfect Peace.